Notes on Exotic Utility Functions

Ref: John Y. Campbell class notes based on
Abel (AER Papers and Proceedings, May 1990)

Habit formation

Abel (1990) gives a simple introduction.
Define ex post utility

$$U_t = \sum_{j=0}^{\infty} b^{j} u\left(c_{t+j}, n_{t+j}\right)$$

Observe $n_t = \left[c_{t-1}^{D} C_{t-1}^{1-D}\right]^{g}$, $g \geq 0$, D≥0. C_t = aggregate consumption

γ=0 : time-separable

γ>0, D=0: "catching up with the Joneses"

γ>0, D=1: habit formation

$$\partial U_t / \partial C_t = U_c(c_t, v_t) + b u_g\left(c_{t+1}, v_{t+1}\right) g \partial v_{t+1} / \partial c_t$$

$$= U_c(c_t, v_t) + b u_g\left(c_{t+1}, v_{t+1}\right) g \partial D_{t+1} / c_t$$

Parameterize $u_c(c_t, v_t) = \left(c_t / v_t\right)^{1-a} / (1-a)$, a=CRRA>0

Then $\partial U_t / \partial C_t = \left[1 - b g D\left(c_{t+1}/c_t\right)^{1-a}\left(v_{t+1}/v_t\right)^{a-1}\right]\left(c_t / v_t\right)^{1-a}\left(1/c_t\right)$

In a representative agent model this simplifies because $v_t = c_{t-1}^{g}$ so we get

$$\partial U_t / \partial C_t = c_{t-1}^{g(a-1)} c_t^{-a} - b g D c_t^{g(a-1)} c_{t+1}^{-a}\left(c_{t+1}/c_t\right)$$

$c_{t-1}^{g(a-1)} c_t^{-a}$ = catching up with Joneses "effect" (? We want γ(a-1)≥0 not γ≥0)

$b g D c_t^{g(a-1)} c_{t+1}^{-a}\left(c_{t+1}/c_t\right)$ = strategic effect: habit (D) reduces $\partial U_t / \partial C_t$

F.O.C. becomes $E_t(\partial U_t / \partial C_t) = b E_t\left[R_{t+1} \partial U_{t+1} / \partial C_{t+1}\right]$

$$\Rightarrow 1 = b E_t\left[R_{t+1} \partial U_{t+1} / \partial C_{t+1} / E_t\left(\partial U_t / \partial C_t\right)\right]$$

This expression is ??? to work with in the habit formation model. In the Joneses model it simplifies:

$$1 = b E_t\left[R_{t+1} \frac{c_t^{g(a-1)} c_{t+1}^{-a}}{c_t^{g(a-1)} c_t^{-a}}\right].$$