A MODEL FOR CONSTRUCTING POVERTY LINES

By N. Kakwani

The Basic Model

A poverty line specifies a society’s minimum standard of living to which everybody in that society should be entitled. Suppose this minimum standard is determined by a minimum utility level $u^*$, which allows the individuals to satisfy their caloric requirements and also their basic non-food requirements such as education, housing, health and so on. Any one whose actual enjoyment of utility is less than $u^*$ is identified as poor. A poverty line is a money metric value of $u^*$.

A poverty line may be defined as consistent, if the minimum standard of living $u^*$ is exactly the same for all individuals irrespective their needs and wherever they live in the country.

The individuals have different calorie requirements because of their age and sex, so the food basket will not be the same all individuals. Similarly, if individuals are living in different regions such as urban and rural areas, their basic non-food requirements will be different. It means that the utility function must take account of these differences.

We may define a utility function as

$$u = u\left[\frac{q_f}{r}, \frac{q_n}{n}\right]$$

(1)

where $q_f$ and $q_n$ are the quantity vectors of food and non-food items of consumption, respectively; $r$ is the calorie requirement of an individual and $n$ is a measure of some other non-food basic needs of that individual. If we fix the minimum standard of living at $u = u^*$, then equation (1) will provide the food and non-food baskets for an individual with given caloric requirement $r$ and basic non-food requirement $n$. The food and non-food poverty baskets will be same for all individuals only if $r$ and $n$ are exactly the same for all individuals. This requirement will never hold, so food and non-food poverty lines will be different for different individuals.

Suppose $p_f$ and $p_n$ are the price vectors of food and non-food items of consumption, respectively, then using the conventional treatment of consumer choice, we maximize the utility function

$$u = u\left[\frac{q_f}{r}, \frac{q_n}{n}\right]$$

(2)

subject to the budget constraint

$$p_f \cdot q_f + p_n \cdot q_n \leq x$$

(3)
where $x$ is the total expenditure or income that is available to the consumer.

This maximization procedure yields the food and non-food demand functions as

$$q_f = r g_f \left(x, rp_f, np_n\right)$$  \hspace{1cm} (4)

and

$$q_n = n g_n \left(x, rp_f, np_n\right)$$  \hspace{1cm} (5)

respectively. These equations are the Marshallian demand functions (Marshall 1930). Substituting (4) and (5) into (3) yields the cost function

$$x = e \left(u, rp_f, np_n\right)$$  \hspace{1cm} (6)

which is the minimum cost of buying the individual utility $u$ at given food and non-food prices.

Further, substituting (6) into (4) and (5) yields the Hicksian food and non-food demand equations (Hicks 1957):

$$q_f = r g_f \left(u, rp_f, np_n\right)$$  \hspace{1cm} (7)

and

$$q_n = n g_n \left(u, rp_f, np_n\right)$$  \hspace{1cm} (8)

respectively.

The food and non-food poverty lines are then obtained by substituting $u=u^*$ in (7) and (8), respectively as

$$F = p_f q_f = r p_f g_f \left(u^*, rp_f, np_n\right)$$  \hspace{1cm} (9)

and

$$NF = p_n q_n = n p_n g_n \left(u^*, rp_f, np_n\right)$$  \hspace{1cm} (10)

Equations (9) and (10) give the food and non-food poverty lines at the point where the individuals enjoy the same level utility $u^*$. These lines will be different for different individuals because of individual differences in calorie requirements and basic non-food needs. The sum of food and non-food poverty lines gives the total poverty line.
Equations (9) and (10) give the utility consistent food and non-food poverty lines. If we know \( u^* \), we can determine both food and non-food poverty lines. The difficult problem is: How do we determine \( u^* \)? The following solution is proposed.

The food poverty line should satisfy the requirement that calorie intake is equal to the calorie requirement. Suppose \( c \) is the vector that converts food quantity vector \( q_f \) into calories. \( c.q_f \) is the number calories that are obtained from the food basket \( q_f \), which should be equal to calorie requirement \( r \). Thus, using (9), we obtain

\[
c.g_f(u^*, r_f, n_p, n) = 1 \quad (11)
\]

This equation should hold for all exogenously determined values of \( r, p_f, n \) and \( p_n \). This means that the function \( g_f(u^*, r_f, n_p, n) \) should not contain \( r_f \) and \( n_p \) as its arguments and should depend only on \( u^* \). The food poverty line in (11) will then be given by

\[
F = p_f q_f = r_f g_f(u^*) \quad (12)
\]

Since the food poverty line can also be written as the product of calorie requirement and calorie cost (which is the expenditure on food per calorie), which from (12) immediately gives calorie cost function as

\[
c_{cost} = p_f g_f(u^*) \quad (13)
\]

which shows that the calorie cost depends on two factors, namely, food prices and the utility \( u^* \). It means that real calorie cost, which is adjusted for prices, is give by

\[
c_{cost}^* = g_f(u^*) \quad (14)
\]

Since \( g_f(u^*) \) is a monotonically increasing function of \( u^* \), it implies from (14) that the real calorie cost is a monotonically increasing function of the utility people enjoy. This proves Lemma 1.

**Lemma 1:** If any two persons have the same real calorie cost at the point where they satisfy the caloric requirements then they will enjoy the same level of utility.

This lemma implies that we can determine the minimum standard of living measured by the utility level \( u^* \) by the real calorie cost. The real calorie cost can be calculated from the data for different quintiles. If for instance we choose the bottom quintile as our reference group, we can use the calorie cost of this group to construct food and non-food poverty lines.

**Food Poverty Line**

The calorie norms are generally available for each country. If there are not available, we can use the FAO norms. These norms are different for different persons because of differences in age and sex. The household income and expenditure surveys provide
information on age and sex of each individual within a household. Given the caloric norms and information on age and sex of each individual, we can easily calculate the per capita caloric requirement of each household.

The food poverty line can be obtained for each household if we multiply the household’s per capita calorie requirement by the calorie cost. In order to maintain consistency of poverty lines, we must use the same real calorie cost for all households. This will ensure that two households will enjoy the same standard of living if their per capita food expenditure is equal to their per capita food poverty line.

The calorie cost that we use must reflect the consumption pattern of the population that we regard to be poor in a given country. Or in other words, we must choose a reference group. We can calculate the calorie cost for different quintiles of the population formed on the basis of per capita household consumption. It may be reasonable to choose the population belonging to the bottom quintile as a reference group. But the choice of reference group should be determined on the basis of commitment the governments want to make in terms of allocating resources to poverty alleviation programs.

Having determined the calorie cost of the reference group at the national level, we need to make an adjustment for difference in regional costs of living differences. Thus, we need to estimate the regional costs of living indices for food items of consumption. These estimates can be easily obtained if we know the average prices of different items of food that are consumed by the population. In the construction these indices, we must use the food basket of the reference group (the population in the bottom quintile). These indices will allow us to estimate separate calorie costs for each region. Multiplying the regional caloric costs by the household’s per capita calorie requirement will immediately give us the per capita food poverty line for each household.

**Non-food Poverty Line**

Suppose we have obtained the food poverty line $F$ on the basis of nutritional requirements. Substituting $F$ into the food expenditure function (derived from the consumer theory), we can solve it for the utility level $u^*$, which will be implied by the food poverty line at the given price vector $p$. Using $u^*$ into the total expenditure function, we can obtain the total poverty line, $z^*$, which will be consistent with the utility level $u^*$. The non-food poverty line will be equal to $z-F$. We describe this procedure using a simple diagram.

In Figure 1, the horizontal axis represents the utility level and the vertical axis represents the expenditures. The figure depicts the food and the total expenditure function, both of which are the increasing functions of the utility level. $C$ is the point that corresponds to the food poverty line on the food expenditure function. Corresponding to point $C$, we obtain $B$ on the x-axis, which gives the utility level $u^*$ that is consistent with the food poverty line.

Corresponding to point $B$ on the x-axis, we obtain point $D$ on the total expenditure function, which gives $BD$ as the total poverty line that is consistent with the utility level.
Obviously then, CD will be the non-food poverty line. The non-food poverty line so obtained will be consistent with the standard consumer theory.

Ravallion (1998) suggested estimating the nonfood poverty line using the idea that if a person’s total income is just enough to reach the food threshold, anything that a person spends on nonfood items will be considered, as basic nonfood needs. According to this idea, the nonfood poverty line is the household’s nonfood expenditure at which the household’s total expenditure is equal to the food poverty line. At this point, the household’s income is just sufficient to buy only the nutritionally adequate food basket so that any expenditure a household incurs on non-food will be absolutely essential.

In the figure, E is the point at which the total expenditure is equal to the food poverty line. At this point, FE will be the non-food poverty line, which will always be less than CD. The non-food poverty line will correspond to the utility level \( _{u^*} \), whereas the food poverty line corresponds to the utility level \( u^* \). Thus, the food and non-food poverty lines do not imply the same level of consumer utility. Thus, we call the Ravallion’s method as inconsistent with the standard utility theory. We recommend to using CD as the non-food poverty line.
This methodology can be used to determine average non-food poverty line separately for each region. This methodology will take account of differences in basic needs in non-food items of consumption in different regions because it determines non-food poverty line in each region that will be consistent with the utility level at the point where caloric needs are satisfied by the households.

The average non-food poverty line can be decomposed into several components, such as clothing and footwear; housing, water, electricity and gas, furnishing and household equipment; health; transport; communication; and education. It is possible that some of the items of non-food poverty line will consist of items such as alcohol, tobacco, leisure and so on. Such items should be excluded from the non-food poverty line so that we keep only the very basic items of consumption.

**Taking Account of Economies of Scale in Non-food Poverty Line**

Since different households have different needs so every household cannot have the same non-food expenditures. In the case of food expenditures, the household needs were assumed to be proportional to per capita calorie requirements. In the case of non-food, we do not have any logical basis for allocating expenditures to each household. In the absence of such a basis, we assume that the non-food needs do not vary with the age and sex of household members. But household do incur economies of scale because of their size. A person living alone will incur more per person expenditure than two persons living together sharing the household goods. This happens because of the public goods that members of the household share without affecting their individual welfare. We may make the following adjustment to take account of economies of scale.

Suppose there are $k$ food components. The mean non-food poverty line $\text{MNFPL}$ is the sum of the $k$ components:

$$\text{MNFPL} = \sum_{j=1}^{k} (\text{MNFPL})_j$$

where $(\text{MNFPL})_j$ is the mean of the $j$th component, where $j$ varies from 1 to $k$. The different non-food components have different degree of economies of scale depending on their degree of sharing. Suppose $\theta_j$ is the economies of scale parameter for the $j$th component of the non-food poverty line. which takes value 1 if the $j$th component is a purely private good and takes value 0 if the $j$th component is a purely public good. Suppose $n_i$ is the size of the $i$th household, then the consumption of the $j$th component by the $i$th household will be given by

$$(\text{NFPL})_{ij} = k (\text{MNFPL})_j n_i (\theta_j - 1)$$

where $k$ is the constant of proportionality. If $\theta_j$ is equal to 1, then every household will be allocated the same per capita expenditure of $(\text{MNFPL})_j$ implying no economies of scale.
for the jth component. If \( \theta_j \) is equal to 0, the ith household will be allocated the per capita expenditure of \((\text{MNFPL})_j/n_i\). The parameter \( k \) is determined so that the mean of \((\text{NFPL})_{ij}\) across all households is equal to \((\text{MNFPL})_j\), which ensures that the adjustment for economies of scale does not change the population mean of the each non-food component. The per capita non-food poverty line for the ith household will then be given by

\[
(\text{NFPL})_i = \sum_{j=1}^{y} (\text{NFPL})_{ij}
\]

To implement the methodology just outlined we need to know the economies scale parameters \( \theta_j \). The estimation of \( \theta_j \) is very difficult. There exists no credible methodology. Lanjouw and Ravallion (1995) estimated the economies of scale using Engel’s model in which the share of the budget devoted to food correctly indicates welfare between households of different size and composition. The main objection against this approach is the implicit assumption that all commodities provide the same degree of economies of scale. Since there are both private and public goods, it is not correct to assume that all goods provide the same economies of scale. We take the view that it is not feasible estimate the economies scale parameter from the consumption patterns of the households.

Although the clothing is generally a private good attributed to individual members of the household, some sharing of clothing does go on within the households. So \( \theta_j \) for clothing may be to be equal to 0.9, which means there is a saving of 10 percent because of economies of scale in clothing. Housing including utilities and furnishing and household equipment are public goods so we may assume \( \theta_j \) for these goods to be equal to 0. The health services can be regarded as a purely private good (because there cannot be sharing of health services) so we may assume the economies of scale parameter for health to be equal to 1. The Households incur expenditure on education only because of presence of children in the household so we assume that expenditure on education is proportional to the number children in the household (divided by household size). Similarly, we may assume that only working adults incur expenditure on transport and communication so expenditure on transport may be made proportional to number of working adults divided by household size.

**Up-dating of Poverty Line**

We should up-date the poverty line so that the minimum standard of living implied by the poverty line remains the same over time. If this requirement is not satisfied, then we cannot make poverty comparisons over time. Over time, number things are changing. For instance, the household size and composition may change over time, which has important implications caloric requirements. The changes in household size affect the distribution of non-food poverty line across households because of economies of scale that occurs within households. The following procedure may be adopted.
First of all we must ensure that the real calorie cost remains the same over time. In order to achieve this, we must know the regional consumer price indices for food, which are generally available in most countries. Since we know the nominal calorie cost in each region in the base year, we can estimate the nominal caloric costs in each region in the terminal year by means of regional consumer price indices for food.

Given the new household survey in the terminal year, we can estimate the per capita caloric requirement for each household. Multiplying the per capita caloric requirement in the new survey by the nominal caloric cost for each region, we will immediately obtain the per capita food poverty line for each household.

To maintain consistency over time, we must ensure that the real average expenditures on various non-food components do not change over time. To achieve this, we will need to know the consumer price indices for each of the \( k \) non-food components. These non-food price indices can then be utilized to estimate the average non-food poverty line by components in the terminal period. The average non-food poverty lines so obtained in the terminal period can then be used to determine non-food poverty line by components for each household by making economies scale adjustment as described above.