Solutions Problem Set 2 - Poverty

Exercise 1

1.1

In a sample survey in which the proportion of poor (P^0) is 30% and the sample size is 25,000 medium-sized families of 4 people. Calculate the 95% confidence interval of P^0.

Solution

Consider a population of size N and a random variable X with mean μ and variance σ^2. We then have that, using the Central Limit Theorem

$$P(\mu - z_{\alpha/2}\frac{\sigma}{N^{1/2}} < X < \mu + z_{\alpha/2}\frac{\sigma}{N^{1/2}}) = (1-\alpha)$$

where $\alpha$is the level of significance and $z_{\alpha/2}$ is the $\frac{\alpha}{2}$ critical value of the Normal distribution. For the 95% confidence interval, we have that

$$P(\mu - 1.96\frac{\sigma}{N^{1/2}} < X < \mu + 1.96\frac{\sigma}{N^{1/2}}) = 0.95$$

Now, note that P^0 can be interpret as a probability and the distribution of individuals under the poverty line can be viewed as a Bernoulli dstribution. We have a sample of $N = 4 \times 25,000 = 100,000$, and each observation follows a Bernoulli distribution with $\mu = P^0 = 0.3$ and $\sigma^2 = P^0 \times (1 - P^0) = 0.21$. Therefore, we have that the confidence interval is

$$CI \approx [0.3 - 1.96 \times \frac{0.46}{316.2}, 0.3 + 1.96 \times \frac{0.46}{316.2}] \approx [0.29715, 0.30285]$$

1.2

Let us say that this society is divided into only two groups X and Z, with respectively 10,000 and 15,000 families each, but with identical average sizes. If the P^0 of X is 30%, what would be the largest size of the P^0 of Z so that the hypothesis that the P^0 of X is equal to P^0 of Z could not be rejected?

Solution

1.3

Now if the Mean Quadratic Poverty Gap (P^2) for the total population and for group X coincides in 10%, what is the P^2 of Group Z and its relative contribution for the total P^2 ?

Solution

Total P^2 can be decomposed as follows

$$P^2 = \frac{N_X}{N} P_X^2 + \frac{N_Z}{N} P_Z^2$$

where P_X^2 and P_Z^2 are the P^2 for group X and Z, respectively. N_X and N_Z are the populations of groups X and Z and N is the total population. Therefore, we have that

$$0.1 = \frac{10,000}{25,000} \times 0.1 + \frac{15,000}{25,000} P_Z^2$$

$$\Rightarrow 0.1 = 0.4 \times 0.1 + 0.6 \times P_Z^2$$

$$\Rightarrow P_Z^2 = \frac{0.1 - 0.04}{0.6} = \frac{0.06}{0.6} = 0.1$$

We have than that P^2 of group Z is 10%. Its contribution for the total P^2 is

$$\pi_Z = \frac{\frac{15,000}{25,000} P_Z^2}{P^2} = \frac{0.6 \times 0.1}{0.1} = \frac{0.06}{0.1} = 0.6$$

That is, the contribution of the P^2 of group Z for the total P^2 is 60%.

1.4

Explain in three steps how to impute rental income to the calculation of social indicators.

Solution

1. Using household surveys and restricting the analysis to the households for which we have data on rental income, we estimate a regression in which the dependent variable is rental income and the independent variables are characteristics of the property (like location, number of rooms, number of toilets, etc.) and individual characteristics of the tenant.

2. Once we have the estimated coefficients, we impute the rental income for the households that are owners of the properties they live.

3. We then recalculate the income for the households that are owners of the property they live, summing the imputed rental income to all other income sources.

1.5

How to incorporate the possibility of different degrees of economies of scale by the size of households into social welfare measures?

Solution

We could use the following concept of income

$$y_{ij} = \frac{Y_{ij}}{I_j N_i^{\theta}}$$

where $\theta \epsilon [0, 1]$ measures the degree of economies of scale, I_{ij} is a deflator for location j and Y_{ij} is total income for household i in location j. Note that the smaller is θ, the bigger is the economies of scale.

1.6

Write the formula and discuss the possible contraindications of the following indicators:

- Sen's Poverty Index
- Decomposition of the effect of inequality on poverty according to Datt-Ravallion (mean and inequality)
- Contribution of category i for the poverty profile of group j using P^2

Solution

- Sen's Poverty Index

$$P_{Sen} = \delta^p P^0 + (1 - \delta^p) P^1$$

where δ^p is the Gini index for the poor.
This index respects the Pigou-Dalton Principle of Transfers. On the other hand, as it uses the Gini, it is not decomposable.

- Decomposition of the effect of inequality on poverty according to Datt-Ravallion (mean and inequality)

Let P be a poverty measure totally characterized by a poverty line z, a mean income μ and a Lorenz Curve Lrepresenting relative inequalities. Its variation can be decomposed according to Datt-Ravallion as following

$$P_{t+n} - P_t = G(t, t+n; r) + D(t, t+n; r) + R(t, t+n; r)$$

where the growth and the redistrivutive components are given by

$$G(t, t+n; r) \equiv P(\frac{z}{\mu_{t+n}}, L_r) - P(\frac{z}{\mu_t}, L_r)$$

$$D(t, t+n; r) \equiv P(\frac{z}{\mu_r}, L_{t+n}) - P(\frac{z}{\mu_r}, L_t)$$

and $R(t, t+n; r)$ is a residual.

This decomposition permits us to identify which are the factors responsible for a variation in a given poverty measure, being possible to determine in which degree the change in poverty was due to a rise in the mean income of the population or a redistribution of the existing wealth. The problem is that this decomposition is not exact, as we can see by the presence of the residual $R(t, t+n; r)$.

- Contribution of category i for the poverty profile of group j using P^2

Let N_{ij} be the population of group j in category i and P_{ij}^2 its quadratic poverty gap. Consider a total population of N with quadratic poverty gap P^2 . We have then that contribution of category $i$of group j for P^2 is $\frac{\frac{N_{ij}}{N} P_{ij}^2}{P^2}$. A policy that focus in a specific group of the population only because it has a high poverty rate, despite being efficient, can have little effect on the total poverty rate if the size of this group is small.

1.7

How to evaluate the impact of a balanced growth process on a given variation observed in two measures of poverty?

Solution

We have to use the Datt-Ravallion decomposition, which permits us to isolate the effect of the balanced growth process from redistributive effects.

1.8

Graphically relate the concept of Second Order Stochastic Dominance and the poverty indicators of the FGT family (P^0, P^1, P^2) .

Solution

Let A and B be two distributions. We say that A dominates B in second order if $P_A^1 > P_B^1$ for every poverty line z. This is equivalent to saying that the poverty deficit curve of A is above the poverty deficit curve of B for every z. It implies that, for $\alpha \geq 1$, $P_A^\alpha \geq P_B^\alpha$. However, we cannot guarantee that $P_A^0 \geq P_B^0$.

1.9

R\$14 per month refers to the minimum monthly amount per Brazilian able to take the income of each miserable up to R\$79 (the line corresponding to 33.3% miserable).

i) What would be the permanent cost of eradicating poverty per Brazilian if the rate of return on social investment was 0.5% p.m.?

ii) If the wage of each non-miserable was R\$7 per hour, how many hours per week would correspond to his contribution?

Solution

i) The permanent cost is given by the resources that we have to collect at a single moment so that, when applied to the rate of return of social investment, it is able to finance the necessary transfers forever without the need for additional contributions. We can find the permanent cost per Brazilian by finding the present value of the transfers by Brazilian, considering the rate of discount as the rate of return of social investment, that is

$$CP = 14 + \frac{14}{(1+r)} + \frac{14}{(1+r)^2} + ... = \frac{14}{1-(1+r)^{-1}}$$

$$= \frac{14}{\frac{1+r-1}{1+r}} = 14\frac{(1+r)}{r} = 14\frac{1.005}{0.005} = 2814$$

ii) Note that R\$14.00 per month would be the contribution of each Brazilian, including miserable and non-miserable ones. What we want to find is the contribution per non-miserable. Let's denote by C_{nm} the contribution per non-misaerable and by C_{tot} the contribution per Brazilian. Let N be the size of the population and Q the size of the miserables. Then, we have that

$$C_{nm} = C_{tot}\frac{N}{(N-Q)} = C_{tot}\frac{1}{(1-\frac{Q}{N})} = 14\frac{1}{1-\frac{1}{3}} = 21$$

The contribution of R\$21.00 per month corresponds to 3 hours of working for the non-miserables.

2.1

Discuss the role of the theta parameter in the formula below:

$$y_{ij} = \frac{Y_{ij}}{I_j n_i^{\theta}},\ \theta \epsilon (0, 1)$$

where family i lives in area j, n_i is the number of people in household i, y_{ij} is the total consumption of family i in region j and I_j is the deflator for area j.

Solution

The parameter θ captures the degree of economies of scale, being the greater the smaller is θ. Consider the case where $I_j = 1$ for every group j and $\theta = 1$. Then, $y_{ij} = \frac{Y_{ij}}{n_i}$. Thus, we are back in the case of per capita income, in which there are no returns to scale. At the other extreme, we can consider $\theta = 0$, and hence we are in the case of total household income.

2.2

Calculate from the data below the minimum monthly cost for the complete alleviation of misery per non-miserable (line R$80.00/month).

- Population: 180
- Monthly Per Capita Household Income: 270
- Proportion of Poor (P^0) : 30%
- P^1 (%) : 20

Assuming a monthly interest rate of 1%, what would be the stock of wealth corresponding to the flow above?

Solution

The proportion of poor (P^0) permits us to find the total number of miserables Q in the total population N

$Q = P^0 \times N = 0.3 \times 180 = 54$

P^1 corresponds to the total cost of erradicating misery measured as a proportion of the poverty line that must be contributed by each individual in the total population. For a poverty line of R$80 and a P^1 of 20%, we have that the contribution per individual is $C_{tot} = 0.20 \times 80 = 16$.

We can then find the contribution per non-miserable necessary to erradicate misery using that

$C_{nm} = C_{tot} \frac{N}{(N-Q)}$

That is,

$C_{nm} = 180 \times \frac{16}{(180-54)} = 22.86$

The stock of wealth corresponding to the flow above and considering a monthly interest rate of 1% is

$\frac{22.86 \times (1+r)}{r} = \frac{22.86 \times 1.01}{0.01} = 2308.57$

2.3

Calculate all Poverty Indexes (P^0, P^1, P^2, $PSen$, etc.) in the sample below assuming a poverty line equal to 3.

{1, 1, 2, 6, 30}

Solution

- P^0

We have 3 individuals below the poverty line, so $P^0 = \frac{3}{5} = 60\%$

- P^1

The cost of erradicating misery is $(3-1)+(3-1)+(3-2) = 2+2+1 = 5$

It corresponds to $\frac{5}{3}$ of the poverty line. Therefore, we have that the contribution for each individual to erradicating misery, or the P^1, is equal to $\frac{5}{3} \times \frac{1}{5} = \frac{1}{3}$.

- P^2

We have that

$P^2 = \frac{1}{N}\sum_{i=1}^{Q}(\frac{z-x_i}{z})^2 = \frac{1}{5}\left[(\frac{3-1}{3})^2 + (\frac{3-1}{3})^2 + (\frac{3-2}{3})^2\right]$

$P^2 = \frac{1}{5}(\frac{2^2+2^2+1^2}{3^2}) = \frac{4+4+1}{5\times 9} = \frac{1}{5}$

- $PSen$

We have that

$PSen = P^0\delta^P + P^1(1-\delta^P)$

where δ^P is the Gini of the poor. To calculate δ^P, we must restrict ourselves to the subsample composed of only the poor individuals, that is, {1, 1, 2}, with $N_P = 3$ and mean $\mu_P = \frac{1+1+2}{3} = \frac{4}{3}$. Therefore, the Gini of the poor is

$\delta^P = \frac{2}{N_P^2\mu_P}\sum_{i=1}^{N_P} ix_i - (1+\frac{1}{N_P}) = \frac{2}{9\times\frac{4}{3}}(1\times 1 + 2\times 1 + 3\times 2) - (1+\frac{1}{3})$

$\Rightarrow \delta^P = \frac{1}{6}$

We have then that

$PSen = 0.60\frac{1}{6} + \frac{1}{3}\frac{5}{6} = 0.38$

2.4

Repeat 3 assuming a balanced growth process of 100%.

Solution

Now the distribution is {2, 2, 4, 12, 60}

- P^0

Now we have 2 individuals below the poverty line, so $P^0 = \frac{2}{5} = 40\%$

- P^1

The cost of erradicating misery is $(3-2)+(3-2)=1+1=2$

It corresponds to $\frac{2}{3}$ of the poverty line. Therefore, we have that the contribution for each individual to erradicating misery, or the P^1, is equal to $\frac{2}{3}\times\frac{1}{5}=\frac{2}{15}$.

- P^2

We have that

$P^2=\frac{1}{N}\sum_{i=1}^{Q}(\frac{z-x_i}{z})^2=\frac{1}{5}\left[(\frac{3-2}{3})^2+(\frac{3-2}{3})^2\right]$

$P^2=\frac{1}{5}(\frac{1^2+1^2}{3^2})=\frac{2}{5\times 9}=\frac{1}{45}$

- $PSen$

Remember that

$PSen=P^0\delta^P+P^1(1-\delta^P)$

where δ^P is the Gini of the poor. Note than the subsample of the miserable is $\{2,\ 2\}$ and therefore the Gini of the poor will be equal to 0. We have then

$PSen=0.4\times 0+\frac{2}{15}\times 1=\frac{2}{15}$

2.5

Make a Datt-Ravallion decomposition of the distribution changes by defining the inequality and growth components above for $\{0,\ 1,\ 2,\ 6,\ 30\}$.

Solution

2.6

Explain the formulas and compare the advantages and disadvantages of the indicators of poverty known as the Mean Poverty Gap (P^1) and the Sen's Index of Poverty (1976). When the two are equal?

Solution

- P^1

$P^1=\frac{1}{N}\sum_{i=1}^{Q}(\frac{z-x_i}{z})$

where N is the size of the population, Q is the number of individuals below the poverty line $z$and x_i is individual income

- Sen's Index

$PSen=P^0\delta^P+P^1(1-\delta^P)$

where δ^P is the Gini of the poor.

P^1 has the property of taking into account the average cost of erradicating poverty, that is, the mean of the differences of incomes of the poor from the poverty line. The Sen's index is a convex combination of P^1 and P^0. P^0 is the less sensible index in terms of taking into account the severity of poverty. Therefore, if $\delta^P\epsilon(0,1)$, we have that the index is less sensible to measure poverty than P^1, which takes into account the distance of the poor individuals from the poverty line and not only the proportion of poor people (P^0). The two measures are equal when the Gini of the poor is equal to 0 (as in Exercise 2.4).

Exercise 3

Comment, agreeing totally, partially or not agreeing and justifying in three or four lines the following propositions (if possible present formulas, graphics or models in capsular forms to illustrate your answer):

i) If the poverty severity curve (the integral of the CDF) of society A is always higher than that of society B, we can ensure that the indicator known as the proportion of the poor (P^0) and the average poverty gap (P^1) are always larger in A than in B.

ii) The Poverty Indicator known as the Mean Poverty Gap (P^1) is higher than the indicator relative to the proportion of poor (P^0) in the design of a system of social goals because it prioritizes the poorest of the poor.

iii) If we adopt the social goal based on the poverty index known as the Mean Poverty Gap (P^1) we will implicitly assume that the first priority is given to the poorest of the poor.

Solution

i) The sentence is false. If the poverty severity curve of society A is always higher than that of society B, we have that A dominates B in third order. This doesn't imply that A dominates B in first and second orders. What we can say is that dominance of first order implies dominance of second order which in turn implies dominance of third order. Therefore, the fact that the poverty severity curve of A is always higher than that of B doesn't imply that P^0 and P^1 are larger in A than in B.

ii) False. Note that in Exercise 2.3, we found that P^0 was higher than P^1. The indicador that prioritizes the poorest of the poor is P^2.

iii) False. The P^1 is the average cost of eliminating poverty. It measures the average distance between the poor individuals from the poverty line, so it doesn't prioritize the poorest of the poor. The indicador that prioritizes the poorest of the poor is P^2.