Social Goals Problem Set: Answer Sheet
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Make a comment, totally agreeing, partially or not. If applicable, justify the following propositions in three or four lines: (if possible, present a formula, or chart in capsular form to illustrate your answer):
1) The consequence of establishing a system in which the greater the poverty, the greater the social investment of the federal government in a region, without any kind of counterpart in terms of results: the final local investment ends up being the same as the case of autarchy. Answer: False. The municipal government spends less in the social area than in non-social spending, it includes additional money in its non-social spending, but additionally reduces its social spending, leveraging the crowding-out effect.

2) The fact that young people are underrepresented in the electoral market makes social spending for this age group less attractive to politicians. Answer: True. Since young people do not vote, the budget of politicians assigned to them is smaller because the short-term return is low and does not generate votes.

3) After the introduction of incomplete information, the poor under the government of the type that is most averse to poverty are as well as they would be with complete information. However, the poor under the government less concerned with the social issue are worse off. Answer: True. Theorem proved in The Design of Social Goals.

4) If the federal government makes unconditional transfers to the municipal government, the situation of the poor does not change, regardless of the utility functions assumed for the federal government and the municipalities. Answer: False, this is true in the case of quasi-linear functions in the budget available and strictly concave in the income of the poor, but we cannot generalize to other cases. Suppose, for example, a case in which the utility function of the municipality is such that its level of utility corresponds to the minimum between the municipality's expenditure and the income of the poor. It is easy to see that, given an unconditional federal transfer, the municipality will allocate additional resources to both general expenses and the poor population.

5) Decentralization of social spending is essential to finance social actions where they are most needed and resources are scarce. Answer: True. The decentralization of federal social spending allows agents who are better informed about the characteristics and needs of the population to be responsible for managing the budget, enabling more effective investments. In the case of the relationship between federal and municipal governments, it is believed that there is asymmetric information, with the latter holding greater knowledge than the former.

6) Conditioning social budget to the assessment of social advances tends to be regressive due to the greater inefficiency of the poor. Answer: False, Progress assessment is one of the few instances where the potential outcomes of the poorest outweigh those of others.

7) A system of social goals a posteriori is indicated in the presence of aggregate shocks. Answer: True, the performance comparison allows accounting for pure aggregate shocks.

8) Even in a situation where the municipality does not have its own money to deal with its social problems, a system of Social Goals can be implemented. Answer: True, a system of Social Goals allows municipalities to allocate resources from other budgets (in this case, from the federal government), regardless of the existence of a sufficient budget for such actions.

9) The structure of incentives provided by social goals contracts is antagonistic with conditional cash transfer programs such as Bolsa-Escola. Answer: False, the structures are similar, since both condition payment to reaching pre-established goals. In the case of social goals, the goals are related to the average income of the poor, while Bolsa-Escola (or Bolsa-Familia) conditions variables such as school attendance and children's vaccination.

10) If we adopt the social goal based on the poverty indicator known as the Average Poverty Gap (P1), we have implicitly assumed that priority is given first to the poorest of the poor. Answer: False, The poverty indicator that prioritizes the poorest of the poor is \[ P^2 = \frac{1}{n} \sum_{i=1}^{q} \left( \frac{Z - y_i}{Z} \right)^2. \] While \[ P^1 = \frac{q}{n} \left( \frac{Z - \bar{y}_p}{Z} \right) \] takes into account the average income of the poor, or rather, the deviation of that income in relation to the poverty line, it does not capture the distributive effects of income among the poor. On the other hand, P2 differentiates the very poor from the little poor.
Social Goals

**Partnership (P)** → Govern knows type $\theta$ of the municipality.

We will now deal with a case in which the federal government acts at the same time as the municipal government in transferring income, just as it used to be in the case of the minimum income program. In addition, this action will take place without setting goals. The difference is that the federal government transfer will be linked to the municipality’s social investment. If it reduces its investment, so does the federal government, and not increase as it was compelled to do in the minimum income program when the municipality’s investment decreased.

What we have, therefore, is the establishment of a partnership between the federal and municipal governments. Since the improvement in social conditions is an externality for both, a natural solution is that the two share responsibility for social investments.

Suppose that for each $T$ transferred by the municipal government, the federal government contributes with a percentage, that is, with $\alpha T$.

1) Set up the problem. The municipality’s problem is Max $G_M + N_p \theta v(Y_p)$ s.a. $G_M + Y_p N_p = Y_M + Y_p N_p \alpha$ and, from the first order condition, we have $v'(Y_p) = \frac{1-\alpha}{\theta}$.

2) Compare the results with the solution with that of an autarchy. **Answer:** In an autarchy, we have $v'(Y_p) = \frac{1}{\theta}$. Since $\alpha > 0$, we know that $v'(Y_p)_{\text{PARTNERSHIP}} < v'(Y_p + T)_{\text{AUTARCHY}}$, and therefore $Y_p^{\text{PARTNERSHIP}} > Y_p^{\text{AUTARCHY}}$. Thus, the income of the poorest is higher in the partnership scenario than in autarchy.

3) Perform sensitivity analysis for different alphas (that is $(\alpha > 0)$). **Answer:** Note that the higher the $\alpha$, the smaller the $v'$, that is, the greater the $Y_p$ transfer. At the limit, where $\alpha$ tends to 1, we have $v'$ tending to zero, that is, the transfers tends to infinity. This is expected, since the federal government will pay for all transfers, leaving the municipality’s budget free to be spent as it wishes. And the municipality is interested in increasing the income of the poor due to the positive externalities generated and captured by the term $N_p \theta v(Y_p)$ in its utility function.

4) While in the situation of unconditional transfers the income remained the same despite the transfers of the federal government, in the case of a partnership between governments we have a worsening in the situation of the poor. Find an explicit solution for the income of the poor based on the following useful function: $v(Y_p) = \sqrt{Y_p}$. **Answer:** In this case, we have $v'(Y_p) = \frac{1}{2 \sqrt{Y_p}}$. The solution in an autarchy requires that $\frac{1}{2 \sqrt{Y_p}} = \frac{1}{\theta}$, which implies $Y_p = \frac{\theta^2}{4}$. In a partnership, we have $Y_p = \frac{\theta^2}{4(1-\alpha)^2}$. 
