*Modeling the Demand for Savings and Consumption in the Permanent Income Hypothesis (PIH) Framework

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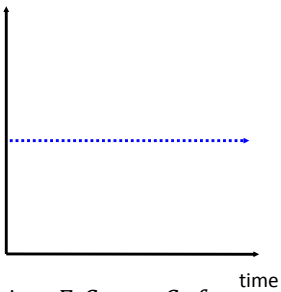
Permanent Income Hypothesis (PIH) - **Hall-Flavin Model**

- This Model is seen as the modern version of the PIH
- **PIH:** $MaxE_t \sum_{i=0}^{\infty}(1+\theta)^{-i}\left(aC_{t+i}-\frac{b}{2}C_{t+i}^2\right)$ s.t. $A_{t+1}=(1+r)(A_t+Y_t-C_t)$
  a) Quadratic and additive utility; i.e. it leads to linear $U'(C_t)$
  b) Single and predictable interest rate and constant at the level of the rate of time preference (r = θ);
  a) Infinite horizon and rational expectations
  - F.O.C (General): $U'(C_t)=E_t\frac{1+r}{1+\theta}U'(C_{t+1})$
    - Given $r=\theta$ and quadratic utility: $C_t=E_tC_{t+1}$
    - By the rational expectations law: $E_tC_{t+2}=E(E_{t+1}C_{t+2})=C_t$ then $E_tC_{t+i}=C_t\ for\ any\ i$



- Integrating the budget constraint:

$$A_t = C_t - Y_t + \frac{A_{t+1}}{(1+r)} = C_t - Y_t + \frac{C_{t+1} - Y_{t+1}}{1+r} + \frac{A_{t+2}}{(1+r)^2} = \sum_{i=0}^{\infty} \frac{(C_{t+i} - Y_{t+i})}{(1+r)^i} \quad \text{where} \quad \lim_{i\to\infty} \frac{A_{t+i}}{(1+r)^i} = 0$$

"Non-Ponzi condition"

- Taking expectations:

$$A_t = \sum_{i=0}^{\infty} \frac{C_t - E_t Y_{t+i}}{(1+r)^i} \quad \text{where } E_t C_{t+i} = C_t = \quad \sum_{i=0}^{\infty} \frac{C_t}{(1+r)^i} - E_t \sum_{i=0}^{\infty} \frac{Y_{t+i}}{(1+r)^i}$$

- Calculating:

$$\sum_{i=0}^{\infty} \frac{C_{t+i}}{(1+r)^i} = C_t \sum_{i=0}^{\infty} \frac{1}{(1+r)^i} \quad \text{(w/ Geometric Progressions formula)} \quad \rightarrow = C_t \frac{1}{1 - \frac{1}{1+r}} = \frac{C_t(1+r)}{r}$$

- Then:

$$C_t = \left(\frac{r}{1+r}\right) A_t + \frac{r}{1+r} E_t \sum_{i=0}^{\infty} \left(\frac{1}{1+r}\right)^i Y_{t+i}$$

  - Capital Income → $\left(\frac{r}{1+r}\right) A_t$
  - Human Capital → $E_t \sum_{i=0}^{\infty} \left(\frac{1}{1+r}\right)^i Y_{t+i}$
  - Human Capital Permanent Income → $\frac{r}{(1+r)} E_t \sum_{i=0}^{\infty} \left(\frac{1}{1+r}\right)^i Y_{t+i}$

How to capture the effects of acquired prospective social Transfers (T_t) on consumption:

$$\frac{r}{(1+r)} E_t \sum_{i=0}^{\infty} \left(\frac{1}{1+r}\right) T_{t+i}$$

What are the impact of Reforms?

Other Hall-Flavin results in differences:

$$C_t - C_{t-1} = \left(\frac{r}{1+r}\right) \sum_{i=0}^{\infty} \left(\frac{1}{1+r}\right)^i (E_t - E_{t-1}) Y_{t+i} \qquad (E_t - E_{t-1}) Y_{t+i} => \theta_i \varepsilon_t$$

Consumption changes (ΔC_t) occurs due to changes in the expectations of futures income. Only changes that were not part of the agent's initial set (t-1) of information would impact consumption. Therefore, the predictive power of other lagged variables besides the lagged consumption would be null. This proposition revolutionized the entire consumption-function industry existing until the mid-1970s (Hall (1978)): Changes in consumption are due to innovations in labor earnings. The later literature combines the above framework with time series models (e.g. ARIMA's, AR( )'s, MA( )'s) for labor earnings. For example, assuming that the income process is an AR(1):

$$Y_t = \phi Y_{t-1} + \varepsilon_t \quad \text{notation: } \phi(L) = 1 - \phi L \text{ , where L is the lag operator} \rightarrow \quad \Delta Ct = \frac{\left(\frac{r}{1+r}\right)\varepsilon_t}{1 - \frac{\phi}{1+r}} = \frac{r}{1+r-\phi}\varepsilon_t$$

The persistence of shocks (income process) **determines their impacts**

Consumption Puzzles

If $\phi < 1$ let's say a stationary process (once if $\phi < 1$ – the shocks effect (ε_t) eventually disappear (so $\phi < 1$) and consumption would be smoother than labor earnings, as Friedman (1957) and Modigliani (1986) highlighted. However, if ϕ tends to 1 then labor earnings become a random walk and the income propensity to consume would be unitary. Meanwhile, if $\phi > 1$ consumption should react more than one to one in relation with labor earnings. Given the existence of permanent shocks in labor earnings, consumption looks too smooth in relation with **income shocks**. This fact constitutes Deaton's (1986) ***excess smoothness puzzle***. Other "puzzle", given the orthogonality tests of Hall (1978) and Flavin (1981), is the ***excess sensitivity puzzle***, which involves the sensitivity of consumption in relation with **expected changes in income.**

Lucas Critique

- **Reference:** Lucas (1976)
- Traditional approach (e.g. Cowles Commission):
- **Model:** $Y_t = G_t B + e_t$ (*)
  - $Y_t =$ endogenous variable (public actions)
  - $X_t =$ exogenous variables (include policy variables)
- **Consumption -** Permanent Income Hypothesis (PIH) of Friedman:
  - **Lesson:** The regression coefficient depends on the income generation process (the independent variable of the problem).
  - **Trap in policy assessment:** What would happen if an estimated consumption function for a period when the income was a random walk were used to predict the effect of a transitional increase in taxation?

The Traditional approach estimates B and uses these estimates for:

a) **Predictions:** $Y_{t+1} = E[G_{t+1}]B$, where $E[G_{t+1}] = G$'s predictions

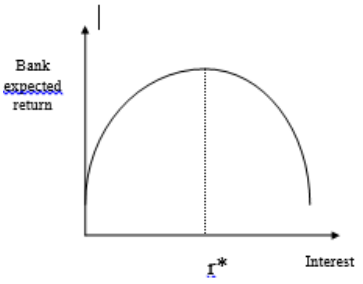
b) **Policies assessment:** Choose different trajectories for $\{G_t\}$ to through (*) choose different trajectories for $\{Y_t\}$

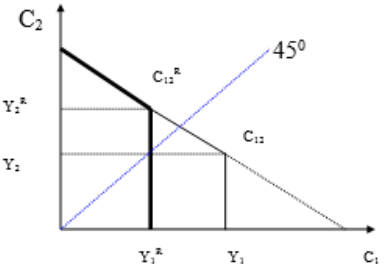
LIQUIDITY CONSTRAINTS

Overview: In terms of consumption/savings decision, credit rationing generates the possibility that agents whose payment capacity is not directly perceived (or warranted) by financial institutions are restricted in the credit market. The credit constraint would to generate a high marginal propensity to consume in relation to available net resources (i.e. current income). Typically, the probability of liquidity constraint to be effective is higher among agents whose wealth is human capital intensive (i.e., workers with an income profile with + Slope and/or impatient agents (in the sense of having a rate of time preference above the interest rate). Liquidity constraint can be seen as a response to arguments of Ricardian Equivalence and rational expectations, in which changes in current income in general would not impact consumption over the same period. In the case of liquidity restricted agents, even temporary changes in current income, for example the result of a tax policy, would affect consumption in a one-to-one relationship.

One way to incorporate credit constraints in framework is through a non-negativity constraint on net assets (i.e.; **At >=** 0). If this constraint is binding (i.e. **At =** 0) then all increases in current earnings will be consumed

The left graph give the basis of equilibrium credit rationing that leads to liquidity constraints. We will look at this model later.





Liquidity Constraints and Goods Indivisibilities

Financial assets accumulation for the acquisition of indivisible assets may result from lack of access to credit when individual monthly income flows are not sufficient to purchase indivisible and high unit value assets such as real estate or entrepreneurial assets. This situation is induced by the existence of rationing and imperfections in the credit market. In this sense, **liquidity constraints could induce greater and not less accumulation of financial assets.** In spite of the fact that by the very definition that **borrowing is negative savings.**

Saving for the purchase of goods is therefore the result of the interaction of two factors: indivisibility of goods and imperfections in the credit market.

Two key related policy instruments are the maximum number of instalments in durables financing or the maximum income share allowed to be used in housing finance.

Interaction of effects also appears in Buffer Stock savings where credit constraints Interact with income uncertainty reinforcing Precautionary savings. Also simple cash flows fluctuations can generate savings motives as in the Life-Cycle model or in the story below

Aggregate Consumption Model with two types of Agents

- **Reference:** Campbell and Mankiw (1989)
- Two types of relevant behavior of individuals in relation to consumption:
  - Agents than consume all their current income (*Keynesian consumers*)
  - Agents that follow the intertemporal model (consumers of *permanent income*)
- **Current aggregate income:** $Ya_t = Yk_t + Yp_t$
  - $Yk_t =$ income appropriated by Keynesian consumers
  - $Yp_t =$ income appropriated by Permanent Income consumers
  - Defining λ as the share of aggregate income that flows to Keynesian consumers → $Y_{at} = \lambda Ya_t + (1-\lambda)Ya_t$
- **Variation in aggregate consumption:** $\Delta Ca_t = \Delta Ck_t + \Delta Cp_t = \lambda \Delta Ya_t + (1-\lambda)\varepsilon_t$
  - $\Delta Ck_t = \Delta Yk_t = \lambda \Delta Ya_t$
  - $\Delta Cp_t = (1-\lambda)\varepsilon_t$, ε = error of forecast

Empirical evidence: Issler et all (1998) estimated λ for 80% of Brazil. This, together with the hypothesis that the restricted individuals are the poorest, would show that 95% of the Brazilian were Keynesians consumers.

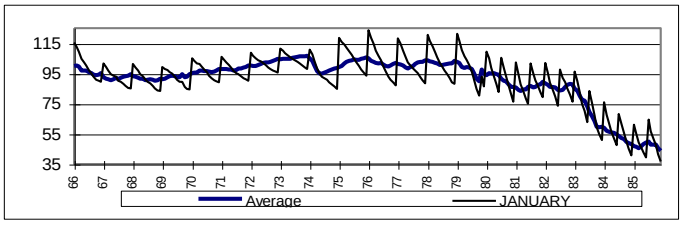
**Aggregate Consumption Model with two types of Agents

The study of aggregate consumption behavior was profoundly altered by the rational expectations revolution in macroeconomics The first example in Robert Lucas's (1976) influential critique of econometric policy evaluation involved consumption. Lucas argued that traditional consumption functions, no matter how well they fit the data were not useful for evaluating the effects of alternative policies. Soon after the Lucas Critique, Robert Hall proposed an approach to consumption founded on the postulate of rational expectations and immune to the problems Lucas pointed out. Hall suggested that aggregate consumption should be modeled as obeying the first order conditions for optimal choice of a single, fully rational and forward looking representative consumer, called the "**Euler Equation Approach**". The paper proposes a simple alternative characterization of the time series data on consumption income and interest rates. It suggests that the data are best viewed as generated not by a single forward looking consumer but two types of consumers **Half the** consumers are forward-looking and consume their permanent income but are extremely reluctant to substitute consumption in response to interest rate movements. **Half of consumers follow the "rule of thumb" of consuming their current income.**

Individual and Aggregate Earnings based Processes

can be very dettached

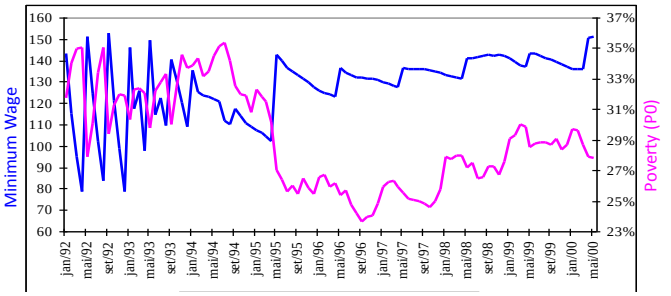
Institutional Wages - Aggregate X Individual -1966-85



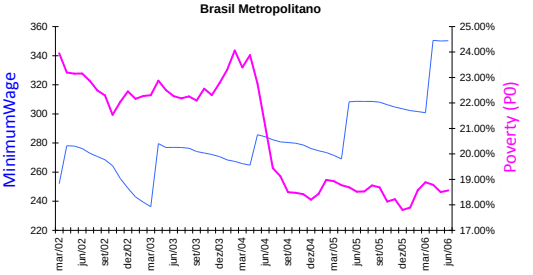
Real Minimum Wage in Brazil 1940 to 2014

Minimum Wage and Per capita Labor Earnings **Poverty (P0)** **PME/IBGE**

Close Inverse Relationship Around Real Plan but gone later

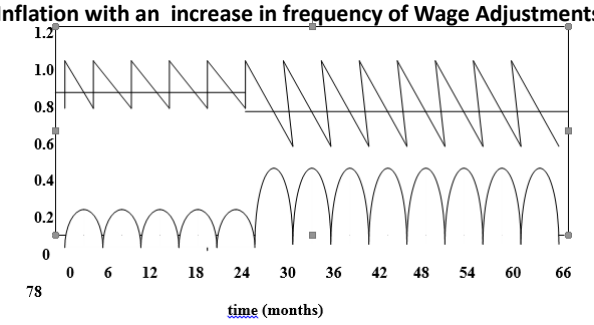


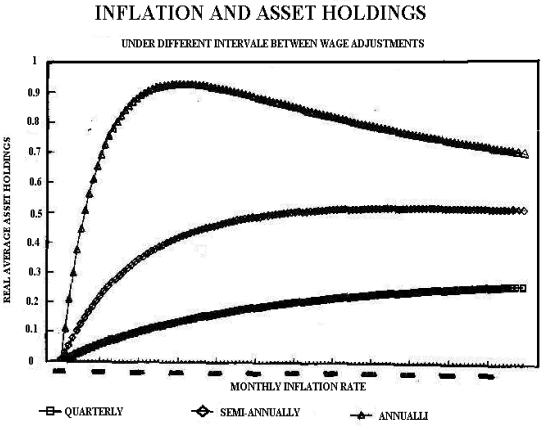
Salário Mínimo Real e Proporção de Pobres (P0)
Brasil Metropolitano



Buffer Stock Savings

Real Earnings, Consumption and Asset Holdings Under Constant Inflation with an increase in frequency of Wage Adjustments

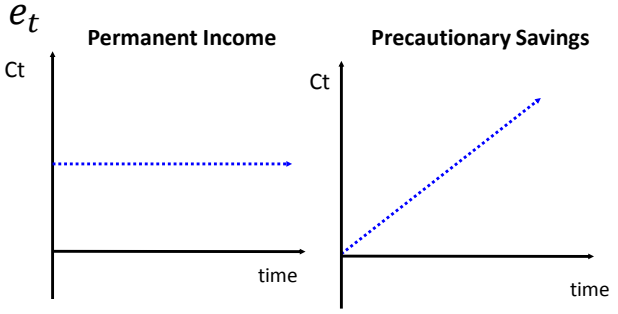




The first effect analyzed above involves the interaction between the declining trajectory of labor earnings between wage adjustments and a smoothing behavior of consumption. Since there is no credit market, the gap between income and expenses of a representative consumer's cash flow takes the form of a high-frequency asset stock. The main result is an average demand for assets that is decreasing with the frequency of wage adjustments, and increasing with the rate of inflation in the relevant range.

Precautionary Savings (due to income uncertainty and U´´´> 0)

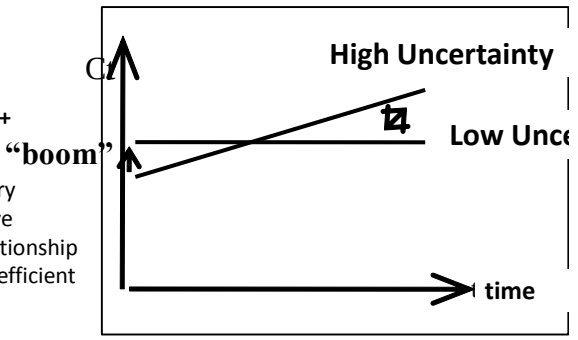
Constant Absolute Risk Aversion (CARA) **Optimal consumption:** $C_{t+1} = C_t + (\alpha\sigma)/2 + e_t$



As time passes uncertainties are solved, individuals allow themselves to consume +

What is the impact of uncertainty on savings?

1. with respect to social security reforms
2. raising the minimum wage
3. Inflationnary uncertainty



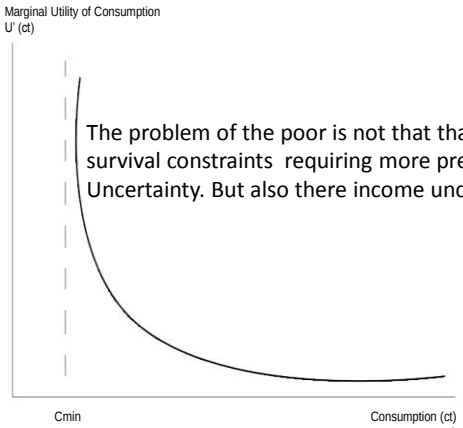
A second additional effect to the last slide argument involves inflationary uncertainty along with a utility function with the third derivative positive reinforces buffer stock savings: if we are willing to accept a positive relationship between level and variance of inflation rates, the expected inflation coefficient in the demand equation for assets will be widened.

Survival Constraint

- **Bliss Level of Consumption**: (C_{min}) minimum level of consumption below which the possibility of survival of individuals is jeopardized

  (just like a poverty line)

- $U(C_t) = \frac{Ln(C_t - C_{min})}{}$

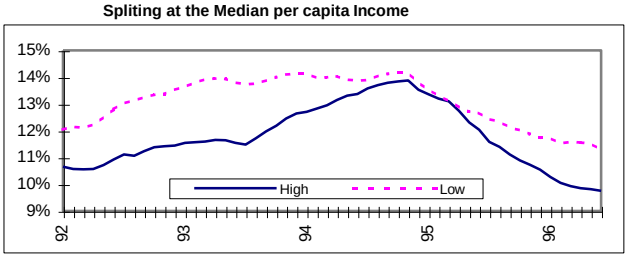


The problem of the poor is not that that they are close to survival constraints requiring more precaution to income Uncertainty. But also there income uncertainty is also greater.

It reinforces Precautionary Motives

Variability of individual monthly earnings over 4 months using longitudinal data PME/IBGE

Spliting at the Median per capita Income



Spliting at the Median Heads Schooling level