Solutions Problem Set Temporal Choice

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Question 1

Comment, agreeing totally, partially or not. If is the case, justify in three or four lines the following propositions (if possible present a formula or graph in capsular form to illustrate your answer):

1.1

One way of explaining excess of demand for credit in equilibrium is to the existence of stickness in capital prices, for example in the case of usury laws.

Solution

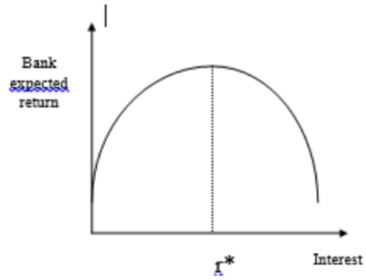
The sentence is false. If prices were totally flexible, in the case of excess of demand for credit we would have an adjustment in the prices, in the sense that they would rise until this excess of demand is zero in equilibrium. Price stickness, therefore, don't let prices to adjust (rise in the case of excess of demand) in such a way that demand and supply are equalized and could indeed lead to excess of demand for credit. However, this would not be an equilibrium because there would still be pressure to change interest rates which could perhaps be materialized in other forms like reducing the level of effective lending by means of compulsory savings in the financial institution, for example. The explanation for credit rationing (excess demand) in equilibrium is provided by the Stiglitz and Weiss model.

1.2

An increase in the interest rate in the Stiglitz and Weiss model always leads to an increase in the bank expected return.

Solution

The sentence is false. Actually, the model predicts that, starting from a low interest rate, increases in it lead to an increase in the the bank expected return up to a certain point, from which it begins to decrease. If we plot the bank expected return against interest rate according to the Stiglitz and Weiss model, the curve will have an inverted u-shape.

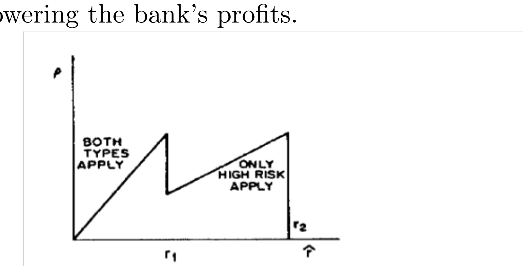


PS: This text proposes an equilibrium (i.e., non ad-hoc) explanation for persistent excess demand in the credit market based on problems of information asymmetry between lenders and borrowers (similarly, the argument could be used to explain persistent excess supply in the labor market). The key-points are that after a certain point, increasing the contractual interest rate decreases the lender's return.

Causes: Adverse Selection (two types of borrowers) and Moral Hazard (two types of actions but one type of borrower).

Lesson: Fixed equilibrium interest rate with excess demand for loans.

The adverse selection aspect of the interest rate is a consequence of different borrowers having different probabilities of repaying their debt. It is difficult to identify "good borrowers" and to do so requires the bank to use a variety of screening devices. The interest rate that an individual is willing to pay may act as one such screening device: those who are willing to pay high interest rates may, on average, be worse risks; they are willing to borrow at high interest rates because they perceive their probability of repaying the loan to be low. As the interest rate rises, the average "riskness" of those who borrow increases, possibly lowering the bank's profits.



1.3

In a competitive equilibrium where economic efficiency is affected by distorting taxes, distributive policies can generate an increase in welfare if they increase borrowers' collateral in a context of perfect information.

Solution

The sentence is false, only because of perfect information. The very existence of information assymetries create inefficiencies in capital markets that can be dealt with collateral. In this way redistributive policies could lead to better working of credit markets through colateral. This situation would break the usual trade off between efficiency and equity but only under imperfect information.

Credit rationing generates inefficiency. Typically, the probability of liquidity constraint to be effective is higher among agents whose wealth is human capital intensive (i.e., workers with an income profile with higher slopes) can be seen as a response to arguments of Ricardian Equivalence and Rational Expectations.

One way to incorporate credit constraints in this framework is through a non-negativity constraint on net assets (i.e., $A_t \geq 0$ for every period). If this constraint is binding (i.e. $A_t = 0$), then redistribution towards the young in our model can lead to increases in both equity and efficiency.

1.4

Microcredit motto: Credit does not create potential opportunities

Solution

The sentence is true. Credit doesn't create potential opportunities indeed but allows that the potential opportunities are used by individuals that without credit wouldn't have the means to invest in them.

1.5

Credit market imperfections (liquidity constraints) can explain why permanent increases in income that were already part of the economic agents' information set produce significant effects on the level of consumption.

Solution

The sentence is true. First, let's consider the case without credit market imperfections. According to the modern life cyle theory of consumption or the modern version of the Permanent Income Hypothesis (PIH) proposed by Hall (1978) through the Euler Equation approach, a permanent increase in income that was already part of the agent's information set implies in a increase in the level of consumption, with no significant effect on current consumption. If we have credit market imperfections like liquidity constraints, the constrained agents won't be able to smooth consumption as they would like and the impact of permanent increases in income as described above will be stronger.

1.6

Friedman's theory of permanent income can explain why families with low current income have higher propensity to consume the current income in comparison with the other families.

Solution

The sentence is true. This empirical regularity inspired the design of the Permanent Income Hypothesis (PIH). Individuals consume according to their permanent income. Low current income individuals have a disproportionately high share of permanent income and a negative transitory income shock that makes current income lower so they tend to consume more than their current income. Think of a negative ϕ in the equation

$$\triangle C_t = \frac{r}{1+r-\phi}\varepsilon_t$$

1.7

In a country with a high inequality level such as Brazil, heterogeneity in consumer's behavior is not properly incorporated by representative agent models.

Solution

The sentence is true. When inequality is very high, such as in Brazil, we have that people are very different in terms of income but also in terms of other characteristics which impact income. Therefore, if this is the case, representative agent models are not very good to capture consumer's behavior because they assume homogeneity in the consumers. High inequality is related to higher heterogeneity in consumer's behavior, which is not captured by representative agent models. For example, Issler et al. (1998) estimated the share of income accruing to liquidity constrained consumers as 80% for Brazil. This, together with the hypothesis that the restricted individuals are the poorest, would show that 95% of the Brazilian were liquidity constrained consumers, while the top 5% would follow the temporal model behavior. From then on the share of credit to GDP rose in Brazil, so this is probably more balanced now.

PS: Aggregate Consumption Model with two types of Agents (Campbell and Mankiw, 1989).

We asssume that there are two types of relevant behavior of individuals in relation to consumption: agents that consume all their current income (Keynesian or liquidity constrained consumers) and agents that follow the intertemporal model (consumers of permanent income). Current aggregate income is defined as

$$Y_{at} = Y_{kt} + Y_{pt}$$

where Y_{kt} is currrent income appropriated by Keynesian consumers and Y_{pt} is current income appropriated by permanent income consumers. Defining λ as the share of aggregate income that flows to Keynesian consumers, we have that

$$Y_{at} = \lambda Y_{at} + (1 - \lambda) Y_{at}$$

Variation in aggregate consumption is given by

$$\triangle C_{at} = \triangle C_{kt} + \triangle C_{pt} = \lambda \triangle Y_{at} + (1 - \lambda) \triangle Y_{at}$$

1.8

According to the precautionary savings model, falls in the variability of income inherent to the stabilization process have a strong impact on the current level of consumption as they relax consumers budget constraint.

Solution

The sentence is partially true. To be more precise, it is true with respect to the final effects but it is false with respect to the channels. Let's consider the following model. The consumer's problem is

$$max \;\; E_t[\sum_{t=0}^{T} (-\frac{1}{\alpha}) exp(-\alpha C_t)$$

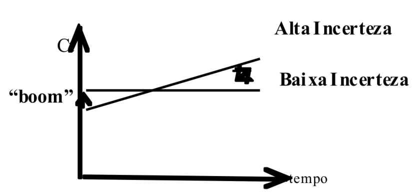
$$s.t \;\; A_{t+1} = (A_t + Y_t - C_t)$$

$$Y_t = Y_{t-1} + e_t \;, \quad e_t \sim N(0, \sigma)$$

In this model, consumers have constant absolute risk aversion given by α. Solving this problem, we have from the First Order Conditions (FOCs) the following Euler Equation

$$C_{t+1} = C_t + \frac{(\alpha\sigma)}{2} + e_t$$

Graphically it can be represented by



Uncertainty does not affect the expected value of the budget constaint (the area under the consumption path across time that represents grapically the

Euler Equation that comes from the FOC of the problem). Different form liquidity constraint, precautionary savings is a self-inflicted constraint. If the slope changes so will the intercept (to keep the area constant).

Its effects operate through the format of the utility function (U"' is positive, which means individuals are prudent) interacting with uncertainty.

Savings would be equal to

$$S_t = [\frac{1}{(T-t)}]A_t + [\alpha(T-t-1)\sigma]$$

Note that the variability of income is given by σ. We see from the equation above that falls in the variability of income imply in a reduction in the current level of savings, which therefore impacts positively current consumption, ceteris paribus. The impact will be stronger the higher is the risk aversion and the time horizon when the current decision is being taken (given by $T-t$) and will be weaker for older individuals (with higher t), ceteris paribus.

1.9

Irrespective of imperfections in the capital market, a greater smoothing of the individual's income between different moments of time and states of nature results in social welfare gains.

Solution

The sentence is false. Only when capital markets are imperfect, income smoothing matters. Otherwise individuals could do that through capital markets.

1.10

Consider the model with ex post utility function given by:

$$U_t = \sum_{j=0}^{\infty} B^j u(c_{t+j}, v_{t+j})$$

where $v_t = [c_{t-1}^D, C_{t-1}^{1-D}]^\gamma$, $\gamma \geq 0$, $D \geq 0$ and C_t is aggregate consumption.

This model is always time-separable and the greater the parameter D, the greater will be the impact of the demonstration effect exerted by the neighbors ("catching up with the Joneses" effect).

Solution

The sentence is false. This model is time-separable only when $\gamma = 0$. Also, the "catching up with the Joneses" effect is higher the smaller is the parameter D, considering that $\gamma > 0$. The effect is maximized when $D = 0$.

1.11

If we incorporate survival constraints in the stochastic consumption model with a CRRA utility function like $U(C_t) = \frac{(C_t - C_{min})^{1-\gamma}}{1-\gamma}$, the demand for savings will increase.

Solution

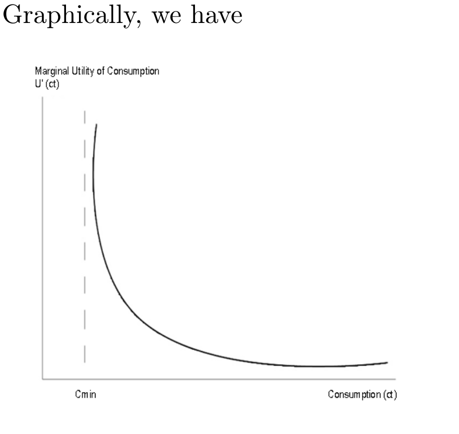
The sentence is true. This specification implies that the optimal level of consumption is greater to C_{min} in every period, that is, $C_t^* > C_{min}$ for every period. To see this, note that

$$U^{'}(C_t) = (1-\gamma)\frac{(C_t - C_{min})^{1-\gamma-1}}{(1-\gamma)} = (C_t - C_{min})^{-\gamma} = \frac{1}{(C_t - C_{min})^{\gamma}}$$

Therefore, assuming the traditional hypothesis that $\gamma \in (0,1)$, we have that,

$$\lim_{C_t \to C_{min}} U^{'}(C_t) = +\infty$$

Graphically, we have



The specification of the model makes precaution even worse, because in every period the consumer would like to consume $C_t > C_{min}$. Note also that $U^{'''}(C_t) > 0$, that is, we have the necessary condition for precautionary savings. Therefore, we have that with this specification demand for precautionary savings will increase.

1.12

Consider the intertemporal consumption model with quadratic and additive utility function, i.e,

$$Max\ \ E_t[\sum_{i=0}^{\infty}(1+\theta)^{-i}(aC_{t+i} - \frac{b}{2}C_{t+i}^2)$$

$$s.t\ A_{t+1} = (1+r)(A_t + Y_t - C_t)$$

This model cannot explain the relation between savings and uncertainity.

Solution

The sentence is true. First, let's see remember some properties of the model: it leads to a linear marginal utility, so the necessary condition for precautionary savings (which is that the third derivative of the utility is strictly positive) is ruled. This specification indeed leads to a third derivative of the utility that is equal to zero. The other necessary condition for precautionary savings is the existence of income risk.

1.13

Consider the income generation process given by the following equation

$$Y_t = \phi Y_{t-1} + \varepsilon_t$$

Denote $\phi(L) = 1 - \phi L$, where L is the lag operator. We have that the greater is ϕ, the greater will be the sensitivity of consumption to changes in income.

Solution

The sentence is true. Using the lag operator, we have that

$$Y_t = \phi L Y_t + \varepsilon_t$$

$$\Rightarrow (1 - \phi L) Y_t = \varepsilon_t$$

$$\Rightarrow \phi(L) Y_t = \varepsilon_t \Rightarrow Y_t = \phi(L)^{-1} \varepsilon_t$$

The sensitivity of consumption to changes in income will be positive correlated to $\phi(L)^{-1} = \frac{1}{\phi(L)}$. Note that the greater is ϕ, the smaller will be $\phi(L)$ and therefore the greater will be $\phi(L)^{-1} = \frac{1}{\phi(L)}$. Therefore, we have that the greater is ϕ, the greter wil be the sentivity of consumption to changes in income. The interpretation is that the persistence of shocks, captured by the error term ε_t, is greater the greater is ϕ.

1.14

Solow Growth and Life-Cycle models produce same effects of savings rates on GDP growth.

Solution

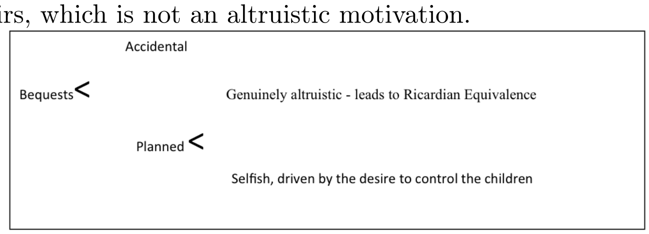
The sentence is false. While in the Solow Growth model savings rates are exogenous, in the Life-Cycle model savings are endonenous. Also, Solow's model predicts that higher savings will lead to higher growth only until a new steady state equilibrium is reached, with higher product and capital. That is, we have that growth is transitory in the case of an increase in the savings rate. In the Life-Cycle model, growth generates higher savings, so the causation between the two models is inverted.

1.15

Making bequests imply necessarily being altruistic.

Solution

The sentence is false. There are strategic bequest motives where the motivation is not altruistic, in the sense of not incorporating the descendents utility function in the utility function of a particular individual. If this is the case, making bequest would reflect ones own utility and therefore wouldn't imply being altruistic. For example, the motivation behind old individuals making bequests to their heirs could be related to an individual desire to control the visits of the heirs, which is not an altruistic motivation.



Question 2 - Discursive Questions

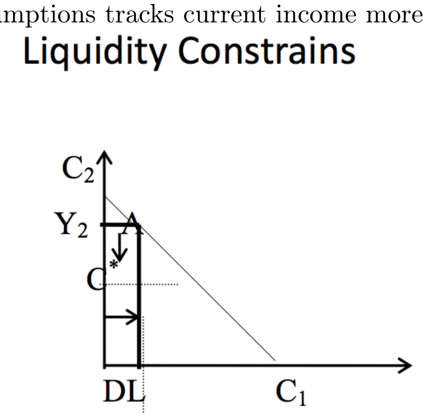
2.1

Discuss the role of the following elements in the explanation why consumption tracks income during the life cycle: liquidity constraints; precautionary savings and habit formation.

Solution

- Liquidity Constraints

If we have liquidity constraints, the constrained agents won't be able to smooth consumption as they would like and therefore consumption would track current income more strongly, reacting positively with increases in income and negatively otherwise. If we didn't have these constraints, individuals would smooth consumption more efficiently, in the sense that in cases of transitory increases (decreases) in current income they would save (borrow) to use (save) in future moments of transitory decreases (increases), so as to keep consumption constant along the life cycle. Therefore, we have that with liquidity constraints consumptions tracks current income more strongly.



- Precautionary Savings

Consider the following model. The consumer's problem is

$$max\ \ E_t[\sum_{t=0}^{T}(-\frac{1}{\alpha})exp(-\alpha C_t)$$

$$s.t\ \ A_{t+1} = (A_t + Y_t - C_t)$$

$$Y_t = Y_{t-1} + e_t\ ,\ \ e_t \sim N(0, \sigma)$$

In this model, consumers have constant absolute risk aversion given by α. Solving this problem, we have from the First Order Conditions (FOCs) the following Euler Equation

$$C_{t+1} = C_t + \frac{(\alpha\sigma)}{2} + e_t$$

and savings would be equal to

$$S_t = [\frac{1}{(T-t)}]A_t + [\alpha(T-t-1)\sigma]$$

Thefore, as people ages t rises and savings falls for both life-cycle and precaution motives. Note that we have uncertainity in income. Note also that

$$U(C_t) = (-\frac{1}{\alpha})exp(-\alpha C_t)$$

$$\Rightarrow U^{'}(C_t) = (-\frac{1}{\alpha})exp(-\alpha C_t)(-\alpha) = exp(-\alpha C_t) > 0$$

$$\Rightarrow U^{''}(C_t) = exp(-\alpha C_t)(-\alpha) < 0$$

$$\Rightarrow U^{'''}(C_t) = exp(-\alpha C_t)(-\alpha)(-\alpha) = exp(-\alpha C_t)\alpha^2 > 0$$

In this model, we have that as people ages uncertainities are solved and individuals allow themselves to consume more.

- Habit Formation

Consider the model in question 1.10, where ex post utility function given by

$$U_t = \sum_{j=0}^{\infty} B^j u(c_{t+j}, v_{t+j})$$

with $v_t = [c_{t-1}^D, C_{t-1}^{1-D}]^\gamma$, $\gamma\epsilon[0,1]$, $D\epsilon[0,1]$ and C_t denoting aggregate consumption.

When $\gamma = 1$, we have what is called (total) habit formation. Note that if it is the case, current utility from consuming in a given period will depend on the consumption for the period plus consumption in the period before. That's the idea behind habit formation.

2.2

Explain the intuition behind equation (1) derived from the model below.

Let's consider the case where absolute risk aversion is constant, so we can solve it explicitly. Suppose that consumer's solve the following problem

$$max\ E_t[\sum(-\frac{1}{\alpha})exp(-\alpha C_t)$$

$$s.t\ A_{t+1} = (A_t + Y_t - C_t)$$

$$Y_t = Y_{t-1} + e_t\ ,\ \ e_t \sim N(0,\sigma)$$

Optimal consumptions satisfies the Euler Equation

$$C_{t+1} = C_t + \frac{(\alpha\sigma)}{2} + e_t \qquad (1)$$

Solution

First, note that another way to write Euler Equation (1) is

$$C_{t+1} - C_t = \frac{(\alpha\sigma)}{2} + e_t$$

The interpretation is that consumption from the current period to the next one would depend positively on the level of absolute risk aversion given by $\alpha$and the variance of the income shocks given by σ, plus an error term with zero mean. The intuition is that the greater the variability of income shocks, the less the consumer would be able to smooth his consumption because of the higher difficulty of predicting the future. Also, we have that this effect is stronger for more risk averse (absolute) consumers.

2.3

Define the "consumption puzzle" of excess sensitivity presented in the empirical literature and give some economic rationale for this result.

Solution

The "consumption puzzle" of excess sensitivity occurs when consumption responds too much to predictable changes in income, differently from what the life cycle theory of consumption predicts (that's why it is called a puzzle). Let's assume, as in exercise 1.12, that we have the following Euler Equation in equilibrium

$$C_t - C_{t-1} = (\frac{r}{1+r})\sum_{i=0}^{\infty}(\frac{1}{1+r})^i(E_t - E_{t-1})Y_{t+i}$$

Note that only surprises matter for consumption changes. However, empirical tests show that lagged information that were already part of the consumer information set like past stock prices actually do impact consumption, which contradicts the modern PIH model predictions. This is the case of excess sensitivity. Many empirical studies actually found that income reacted significantly to changes in current income, in contrast to the predictions of the theoretical models. One possible explanation could be the presence of liquidity constraints.

For the sake of completeness, another puzzle found in the empitirical literature is is the excess smoothness puzzle, which is related to unexpected shocks. Let's assume the following income process

$$Y_t = \phi Y_{t-1} + \varepsilon_t$$

Then we have that

$$C_t - C_{t-1} = \triangle C_t = \frac{(\frac{r}{1+r})\varepsilon_t}{1 - \frac{\phi}{1+r}} = (\frac{r}{1+r-\phi})\varepsilon_t$$

If $\phi < 1$ (that is, the income process is stationary), consumption would be smoother than labor earnings, as Friedman (1957) and Modigliani (1986) highlighted. However, if $\phi \to 1$, then labor earnings become a random walk and the income propensity to consume would be unitary. Meanwhile, when $\phi > 1$, consumption should react more than one to one in relation with labor earnings.