Exercises with solutions

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Exercise 1 - Conceptual Questions

Evaluate if each sentence is partially or totally True or False and comment why. Indicate within each small letter items whether correct and wrong elements coexist.

A) Inequality and Welfare

i) a. Strictly speaking, GDP per capita is not a measure of welfare because it does not take into account the level of inequality in the society. b. The existence of economies of scale in households suggests that per capita family income concept is not a good tool to capture poverty. c. The Theil-T Index dual can replace with some advantages the Gini index in the Social Welfare Indicator proposed by Amartaya Sen.

Solution a. The sentence is false. Despite not incorporating inequality, GDP per capita can be used as a measure of welfare. We think in a context of a social welfare function like the one proposed by Sen but with zero inequality, where GDP per capita would capture income in the society and therefore could be welfare.

b. The sentence is true or at least partially true. The existence of economies of scale implies that bigger households have advantages over smaller ones. If we use per capita household income, we are not considering the effect of economies of scale of bigger households. As in general poor households have more individuals than richer ones, we have that not taking into account the effect of economies of scale would overestimate poverty. However, per capita family income concept could be a good tool to capture poverty in many situations.

c. The sentence is true. Compared to the Gini, the dual of the Theil-T has the advantage of being fully decomposable. In addition, it ranges from 0 to 1 like the Gini and therefore it is also easy to interpret.

ii) a. If income inequality captured by household surveys such as the PNAD underestimates actual income inequality by not capturing the income of the

richest, then the level of Brazilian social welfare would necessarily be overestimated by the PNAD. b. Measures of inequality using per capita income tend to underestimate by construction inequality between individual incomes. c. Inequality measures taking into account longer income measurement periods tend to be larger than for shorter periods. d. Absolute inequality measures tend to increase in the case of economic expansion.

Solution a. The sentence is false. It is true that household surveys such as PNAD underestimates actual inequality, but the traditional social welfare measurement incorporates two components, inequality and mean income. Therefore, we cannot say that social welfare is overestimated by the PNAD without taking into account the actual mean income level of the society, which as well as inequality is not totally captured by the PNAD.

b. The sentence is true. By construction, when we measure inequality using per capita household income, we are assuming that there is a "socialization" of the income inside the household, which is a somewhat strong assumption. Actually, we have that in many cases income is not equally shared inside the household and therefore when we use per capita measures we could be underestimating the effect of the inequality between the individuals within a household.

c. The sentence is false. Income measurements taking into account shorter periods usually have more variance than longer ones. Therefore, we usually have that inequality measures taking into account shorter periods are higher than the ones taking into account longer periods.

d. The sentence is true. In the case of a balanced economic growth, we would have that absolute inequality will increase because the same percentage increases are different in absolute terms, as a percentage increase for the richest will correspond to a much bigger increase in absolute terms than the same percentage increase for the poorest. Therefore, we can say that absolute inequality measures tend to increase in cases of economic expansions. However, we could have a situation of economic growth where the increases for the poorest were much larger in percentage terms than the increases for the richest.

iii) a. The inequality index known as J-Divergence makes it possible to directly capture the shares of income groups and individuals in inequality including the richest and the poorest in society, even those with zero incomes. b. Alternatively, the simple inequality indicator corresponding just to the proportion of the 10% richest in income allows to accurately capture the contribution of the between and within groups components in total inequality. c. The proportion of the richest 10% in income and the J-Divergence follows the principle of transfers (Pigou-Dalton).

Solution a. The sentence is false. The J-Divergence corresponds to the sum of the Theil-T and the Theil-L indexes. The Theil-L index has the disadvantage of not being defined for null incomes because of its logarithmic form. Therefore, the J-Divergence also doesn't incorporate individuals with zero incomes.

b. The sentence is false. This indicator doesn't allow us to decompose changes in total inequality into changes between and within groups components, as we are aggregating the individuals into 2 groups as if each group is a representative agent. Therefore, because we are implicitly considering "socialization" of income within each group, we would only be capturing the between groups inequality component, without being able to calculate the within groups (it is implicitly assumed to be zero).

c. The sentence is false or partially false. If we have a transfer from an individual that is part of the 10% richest to another one that is not in this group, we will have that the inequality measure taking into account the proportion of the 10% richest in income will fall once the mean will be the same but the share of the 10% richest will fall. However, if this transfer involves two individuals that are not part of the 10% richest or two individuals that are both part of the 10% richest, we will have no change in the inequality measure. Therefore, we cannot say that this measure follows the principle of transfers. The J-Divergence, in turn, follows the principle of transfers as it is a sum of the Theil-T and the Theil-L indexes and they follow the principle.

iv) a. In the context of the Mincerian (log-linear) earnings equation and its descriptive statistics, the concept of the gross contribution to inequality is measured by the R2 of a regression with only one constant and the variable that we want to evaluate the contribution (e.g education). b. The same regression can yield, the determination of the net contribution of education to inequality. c. The coefficient of education in the mincerian regression gives us how much the absolute change in average schooling affects the income of individuals in absolute terms (in monetary units).

Solution a. The sentence is true. If we run a simple regression with only a constant and the variable we want to evaluate the contribution (in our case education), the R2 will in fact measure the gross contribution to the dependent variable, capturing the variation in income that is explained by the variation in education, or the gross contribution of education to changes in income.

b. The sentence is false. If we want to measure the net contribution of education to changes in income or to inequality, we need to run two regressions, one including as regressors schooling and other relevant variables that affect earnings (e.g gender, age, experience, etc.) and another one without schooling, The difference of the R2 of the first regression and the second one will be the net contribution of education to inequality.

c. The sentence is false. The coefficient of education in the mincerian regression gives us how much on average the absolute change in average schooling affects income in relative terms. It corresponds to the average change in income in percentage terms of one more year of schooling, which is also referred to as the return to education. For example, a coefficient of 0.10 indicates that on average one year of schooling increases income in 10%. This property is a consequence of the log-linear form and the coefficient of the regression is also referred to as

semi-elasticity.

v) a. Simple Lorenz curves enable us to compare directly levels of social welfare between societies. b. Growth incidence curves do not allow us to capture the effect of inequality. c. Concentration ratios share the same range as the Gini index.

Solution a. The sentence is false. Lorenz curves enable us to compare levels of inequality between societies in some cases. In particular, if we have Lorenz dominance (a situation where the Lorenz curve of one distribution is always above the curve for another one), we would be able to say unequivocally which society is more unequal (the one with the Lorenz curve always above the other). However, the level of social welfare of a society will also depend on its average income and this effect is not captured by the Lorenz curve. To compare directly levels of social welfare between societies, we could use Generalized Lorenz curves.

b. The sentence is false. The growth incidence curve allows to see the joint effect of inequality and mean changes.

c. The sentence is false. Concentration ratios range from -1 to 1 while the Gini ranges from 0 to 1. The intuition is that if a program is totally pro-poor, that is, if it benefits the poorest individual in the society, it will have concentration ratio of -1. On the opposite extreme case, it will have concentration ratio of 1.

B) Poverty

i) a. Poverty measures changes can be decompose exactly into mean and inequality components, allowing exact decompositions only in terms of changes in these two components. b. The Theil-T index is an indicator that is not very sensitive to changes in the basis of income distribution, and in this aspect is not very useful for analyzing the relationship between inequality and poverty. c. Regional price indices in general do not affect inequality measures.

Solution a. The sentence is false. We can decompose changes in poverty in terms of changes in mean and inequality, however the decomposition is not exact. The Datt-Ravallion decomposition has a residual term in adittion to the terms corresponding to the mean and inequality components.

b. The sentence is true. The Theil-T index is not sensible to marginal changes in the bottom of the income distribution. The turning point of inequality or the percentil for which a marginal increase in income induces a increase in inequality is very high for the Theil-T index (87th percentile in the case of per capita income for PNAD 2014), being the highest compared to the other traditional indexes.

c. The sentence is false. While temporal price indexes affect only the mean, regional price indexes may affect inequality measures if they are higher (or lower) in poorer regions, for example.

ii) a. If we adopt a social goal system based on the poverty indicator known as the poverty gap (P1) we will implicitly assume that priority is given to the poorest of the poor. b. A given fixed and unique target date embedded in social goals can lead to opportunistic behaviors of public policy managers. c. If we assume a perfectly uniform income distribution, the cost of eliminating poverty will be lower for the Quadratic Poverty Gap (P2) indicator than for the poverty indicator known as the Poverty Gap (P1).

Solution a. The sentence is false. The P1 or mean poverty gap captures the average cost of eliminating poverty by taking the mean of the distance between the incomes of the poor individuals and the poverty line. Therefore, it doesn't give priority to the poorest of the poor. The indicator that gives priority to the poorest of the poor and is inequality sensitive is the P2 or quadratic poverty gap.

b. The sentence is true. We could have that a given fixed and unique target date could imply perverse incentives to the policy makers in the sense that they don't care about how effectively some policy is being implemented, but only about some final result. This could lead to opportunistic behaviors and lack of commitment of public policy managers with intermediate goals, that are extremely important in many cases.

c. The sentence is false because the cost of eliminating poverty is given by the poverty gap or the P1. One can show that $P^2 \leq P^1$. You get points here: let's remember the formulas for the two measures. Denoting N and Q as the total and poor population, z as the poverty line and x_i income for individual i, have that

$P^1 = \frac{1}{N}\sum_{i=1}^{Q}(\frac{z-x_i}{z})$

$P^2 = \frac{1}{N}\sum_{i=1}^{Q}(\frac{z-x_i}{z})^2$

Now consider the case where $x_i = \bar{x}$ for every individual and let's suppose that everyone is poor (otherwhise everyone will be above the poverty line and we would have $P^1 = P^2 = 0$). Note that each term $z - \bar{x}$ lies between 0 and z. Therefore, we will have that $P^2 < P^1$ in this case.

iii) a. Poverty analysis is necessarily arbitrary because it starts from a somewhat arbitrary value for the poverty line. b. Poverty lines are less arbitrary than indigence lines. c. The poverty dominance analysis allows to reduce the arbitrariness derived from choosing a specific poverty line.

Solution a. The sentence is true. The choice of calories intake, the Engel coefficient (see below), the choice of deflators (Laspeyres and Passche) and the data set used, as well as others, depend on judgement values.

b. The sentence is false. Poverty lines may be seen as arbitrary (and even more arbitrary) than indigence lines. This is because it includes the indigence line in its computation plus an Engel coefficient (1/foodshare) which is also subject to arbitrary choices. For example, you may pick a particular reference group for that Engel coefficient.

c. The sentence is true. Poverty dominance implies that certain distribution has higher levels of poverty than another one for any poverty line. Therefore, the arbitrariness from choosing a specific line is reduced.

iv) a. If the poverty severity curve (the integral of the Cumulative Distribution Function (CDF) of income) of society A is always above that of society B, we can ensure that the indicator known as the proportion of poor (P0) and the mean poverty gap (P1) are always higher in A than in B, for any poverty line. b. The non-linearity of the Quadratic Poverty Gap (P2) indicator compromises its decomposition among groups of the society. c. Poverty targets based on P2 have difficulty in inducing actions aimed at the poorest.

Solution a. The sentence is false. If the poverty severity curve of A is always above than B, we have third order dominance (TOD). This doesn't imply that the proportion of poor and the mean poverty gap of A are always higher in A than in B, which correspond to first order dominance (FOD) and second order dominance (SOD), respectively. We have is that FOD implies SOD which therefore implies TOD but we cannot say that the reverse is true. That is,

$FOD \Rightarrow SOD \Rightarrow TOD$,

but the reverse is not necessarily true.

b. The sentence is false. The Quadratic Poverty Gap (P2) indicator is exactly decomposable between groups. That is why it can replace the Sen's Poverty indicator, which is not exactly decomposable because of the Gini of the poor incorporared in it.

c. The sentence is false. P2 is the most pro-poor of the three indicators seen in the course in the sense that it giver priority to the poorest of the poor. Therefore, poverty targets based on P2 induce actions aimed at the poorest of the society.

v) a. Minority groups with low incomes such as indigenous people, although poor, tend to have a low contribution to total poverty. b. In this case, the size of the budget for programs to alleviate poverty should not be guided solely by the rate of poverty incidence. c. Universal income transfer programs for all citizens is more indicated in more unequal countries such as Brazil than in more egalitarian societies.

Solution a. The sentence is partially true. In the case of indigenous people, although poor they represent a small fraction of the society, and therefore will contribute little to total poverty. However, we could have that some minority groups can contribute significantly to total poverty.

b. The sentence is true. The rate of poverty incidence doesn't capture the severity of the problem because the individual contribution of the poorest of the poor and of a poor individual closer to the poverty line will be the same. Therefore, programs depending solely on the rate of poverty incidence would not focus on the poorest of the society. The policy makers will also have incentives

to focus on the individuals closer to the poverty line in the cases where the social targets are aimed at reducing solely the rate of poverty.

c. The sentence is false. In more unequal societies, we have that universal income transfer programs are not indicated. Unequal countries such as Brazil should have income transfer programs that take into account the individual level of poverty, that is, how much income is needed for each individual to alleviate poverty, or individual differences in income. These programs should give more income to the ones that need more and therefore shouldn't be universal.

C) Social Targets i) a. A system where the greater the initial poverty the greater the social transfer from the federal government to a region leads to an increase in non-social spending. b. The final social outcome is superior to the autarky case. c. Such a system should not be applied in a sequence of periods.

Solution a. The sentence is true. In this case, the local government includes the adittional resources in its non-social expenditures but aditionally reduces its social spending, leveraging the crowding-out effect.

b. The sentence is false. As we explained in (a), the crowding-out effect is leveraged in this case and the local government reduces its social spending to a level that is lower than under autarky. Therefore, the final social outcome will be inferior to the autarky case.

c. The sentence is true. This type of system gives local governments incentives to spend less on reducing poverty than in the autarkic cause and could have a negative result in terms of the impact on poverty. Therefore, it should not be applied regularly. In the case of the HIPCs (Highly Indebted Poor Countries), for example, it may create an extra incentive for keeping people poor as a way to access external resources.

ii) Politicians are less concerned with underrepresented groups in the electoral market, such as children. a. There is an impact on investment in education but there is no difference in the performance of poverty by age groups. b. There is nothing to be done about political favoritism in the context of a system of social goals. c. One can eliminate completely the distance of the treatment given to different groups.

Solution a. The sentence is false. The fact that young people don't vote implies that politicians allocate less money to them because the short-term return is low as the investment on the young doesn't generate votes immediately. We could have that politicians allocate less money to reduce poverty among younger groups, preferring to spend this money on social investment for older groups that vote.

b. The sentence is false. A system of social goals based on education as well as poverty and other indicators among specific age groups vulnerable to the problem of political favoritism could have positive results in terms of improving their conditions.

c. The sentence is false. One can eliminate only partially the distance of the treatment given to different groups. See the equation in the slides.

iii) A system of targets based on international indicators such as MDGs and SDGs allows to: a. lengthen the planning horizons of policy makers; b. generate a common ground for actions between different levels of government; c. create automatically insurance against idiosyncratic and systemic shocks.

Solution a. True. The fact that the targets last more than political mandates (25 years in the case of the MDGs and 15 years in the case of the SDGs) may help to mediate and coordinate short run decisions into long run outcomes.

b. True. The neutrality of international targets may help the dialogue between federal, state and municipal levels. They may not be seen as the imposition of governments from possible different political parties over the others.

c. At least partially false. It does not deal with idiossincratic shocks. We need to have an insurance market for that. On the good side the systemic (or aggregated) risk which is in principle not insurable is well dealt within such a system of social targets.

Exercise 2 - Empirical Questions

A) Labor Decomposition: Consider the labor decomposition of individual income for the Active Age Population (AAP or PIA in Portuguese) between the quarters presented below:

Active Age Population (PIA)					
Year - Quarter	Total Labor Income (R$)	Positive Labor Income per Years of Schooling	Years of Schooling	Occupation Rate in the Active Economic Population (%)	Participation Rate in the Labor Market (%)
2015-02	1131,26	181,91	9,89	88,49	71,09
2016-02	1063,04	173,38	9,95	86,1	71,59
Annual Rate of Variation (%)	-6,031	-4,689	0,622	-2,701	0,703

i. What is the actual level of unemployment? b. What is the impact in mean income of the rise in the unemployment? (Hint: Economic Active Population (EAP) = Occupied + Unemployed and Participation Rate = EAP/Active Age Population)

ii. a. If we assume a growth rate for the PIA of 0.5% per year as a result of the current demographic transition, what should be the growth of labor income for the total population? b. Compare the impacts on total income of the demographic bonus with the impact of the rise in average years of schooling of the occupied (educational bonus).

iii. Show how the labor decomposition is constructed in levels and then in rates of variation. (Hint: see the first line of the table and depart from the earnings mass in the Active Age Population)

Solution

i) a. The rate of unemployment in the Economically Active Population is $1 - \frac{ocup}{PEA} = 1 - 0.861 = 0.139$

That is, the rate of unemployment is 13.9%.

PS: We also know rate in the labor market corresponds to $\frac{PEA}{PIA} = 0.7159$. Therefore, we can calculate the rate of unemployment in the PIA as following

$\frac{unemp}{PIA} = \frac{unemp}{PEA} \times \frac{PEA}{PIA} = 0.139 \times 0.7159 \approx 0.1 = 10\%$

That is, the rate of unemployment in the PIA is approximately 10%.

b. The impact in mean income of the rise in unemployment is -2.701%, as on the table.

ii) a. Let's remember the following relation

$totincome = \frac{totincome}{laborincome} \times hourlywage \times educ \times worktime \times \frac{ocup}{PEA} \times \frac{PEA}{PIA} \times \frac{PIA}{pop}$

Considering the variation from 2015-02 to 2016-02, we have that the growth rate of total income for the period is the product of one plus the growth rate of each component above, as we have on the table. Considering a growth rate of 0.5% per year in the PIA, we have that growth rate in total income will be

$(1 - 0.06031)(1 + 0.005) - 1 = 0.94439 - 1 = -0.05561 = -5.56\%$

Alternatively, we can use the sum of the growth rates in a logharitmic approximation. We have

$-6.031\% + 0.5\% = -5.531\%$

b. The total impact of the demographic bonus is 0.05% per year. In terms of shares in the growth rate of total income, we get from the logarithmic approximation that the impact is $\frac{0.5}{5.561} = 10.9\%$. The impact of the rise in average years of schooling of the occupied is 0.622% per year or $\frac{0.622}{5.561} = 11.1\%$ in terms of shares of total population income change, following the same process. Therefore, the impact of the educational bonus is higher than the impact of the demographic bonus.

iii) In terms of levels with respect to the total population, we have that
$totincome = \frac{totincome}{laborincome} \times hourlywage \times educ \times worktime \times \frac{ocup}{PEA} \times \frac{PEA}{PIA} \times \frac{PIA}{pop}$
Therefore, the decomposition in rates of variation is
$(1 + \triangle totincome) = (1 + \triangle \frac{totincome}{laborincome}) * (1 + \triangle hourlywage) * (1 + \triangle educ) * (1 + \triangle worktime) * (1 + \triangle \frac{ocup}{PEA}) * (1 + \triangle \frac{PEA}{PIA}) * (1 + \triangle \frac{PIA}{pop})$
In logarithmic approximation, we have
$\triangle totincome \approx \triangle \frac{totincome}{laborincome} + \triangle hourlywage + \triangle educ + \triangle worktime + \triangle \frac{ocup}{PEA} + \triangle \frac{PEA}{PIA} + \triangle \frac{PIA}{pop}$

B) Difference in Differences: i. Discuss the evolution of the income differential between rural / urban (category omitted) in Brazil between 2001 (category omitted) and 2009 using the regression below in which the dependent variable is the log of income. We just show the interaction term and their respective terms without interaction. Is this movement equality enhancing?

ii. How to interpret the four coefficients in bold of the mincerian model below?

iii. Show the basic formula of the difference in difference estimator and describe its applications.

iv. If the growth rate of the mean rural income is 3% per year, how long would take to double it?

Estimated Coefficients		
Parameter	Estimator	t stataistic**
Rural	-0.88	-118
Year 2009	**0.1**	**19**
Rural*2009	**0.14**	**13**

PS: i. We omitted other controls for macro-region, schooling and gender plus the intercept term

ii. The absolute value of the t-stat should be greater than 1.96 for statistical significance of the coefficients (95% level).

Solution i. The estimated coefficient for the interactive dummy Rural*2009, which is 0.14, indicates that increase in the income for the rural area between 2001 and 2009 was higher than for the urban area. Now let's consider the estimative of the coefficient for the dummy Rural, which is -0.88. It indicates that the income for the rural area was lower than for urban. Analysing the two

coefficients together, we can say that the income differential between the urban and the rural reduced from 2001 to 2009.

ii. The estimated coefficient for the year of 2009, which is 0.1, indicates that income increased approximately 10% between 2001 and 2009 in Brazil. As explained in item (i), the coefficient for the interaction between rural areas and the year of 2009 indicates that income for these areas increased approximately 14% more than for urban areas in the period of 2001 to 2009. The respectives t-statistics are very high and indicates that these estimatives are statistically significant at all usual confidence levels.

iii. The basic formula for the diff-in-diff estimator is

$g_3 = (y_{2,t} - y_{1,t}) - (y_{2,c} - y_{1,c})$

where t indicates treatment group, c control group and the number subscripts represent two different moments in time (1 before the treatment and 2 after the treatment).

In order to study the impacts of local infrastructure policies between two groups, we need data at least for two moments in time. Therefore, the sample is four fold. The interactive effect between the treatment group dummy and the time dummy gives us the difference-in-difference estimator g_3. Once the estimator is obtained, the impact of the natural experiment on the variable to be explained is determined.

iv. Let's consider a growth rate of the mean rural income of 3% per year. Let's denote y_o mean rural income in the baseline. In the following year, income will be $y_1 = y_0(1 + 0.03)$. Following the same logic, we have that $y_2 = y_1(1 + 0.003) = y_0(1 + 0.03)(1 + 0.03) = y_0(1.03)^2$. In general, we have that $y_T = y_0(1.03)^T$. We want to find the value of T such that $y_T = 2y_0$. That is, such that

$2y_0 = y_0(1.03)^T \Rightarrow 2 = (1.03)^T$

If we take logs on both sides, we have that

$ln(2) = T \times ln(1.03) \Rightarrow T = \frac{ln(2)}{ln(1.03)} \approx \frac{0.69}{0.03} = 23$

That is, it would take 23 years to double mean rural income if we consider a growth rate of 3% per year.

Exercise 3 - Quantitative Questions

A) i. What are the possible constraints imposed on the social welfare function below so that the Principle of Transfers (Pigou-Dalton) is observed? ii. Write down the functional form of the Social Welfare function associated with the Gini Index from the equation below. Explain each component. iii. Indicate the key steps following Atkinson's approach by Sen to derive the Gini index. iv. Bonus item (additional 0.5 point): show in broad steps how to decompose the rate of change in welfare from changes in growth and inequality.

$$u(x^*) = \int_0^\infty w(x)u(x)f(x)dx$$

Solution i. We could observe the Pigou-Dalton Principle of Transfers if we assume utility functions $u(x)$ with decreasing marginal utilities or if we impose bigger weights $w(x)$ to the poorest. One special case is if we take $u(x) = log(x)$ and $w(x) = 2[1 - F(x)]$, where $F(x)$ is the cumulative distribution function.

ii. This corresponds to the case where $u(x) = x$ and $w(x) = [1 - F(x)]$.

iii. A Social Welfare Function (SWF) $W = V(x_1, x_2, ..., x_N)$ corresponds to a sum across individuals. Properties of a SWF V are:

- Pareto Optimum – V is increasing in its arguments. If one gets better and nobody worse it increases.
- Symmetry or Anonymity – V depends on individual welfare levels and not on their identity.
- Principle of Transfers (Pigou–Dalton) – For a given total $x = (x_1, x_2, ..., x_N)$, V will be at its maximum point when inequality is at its minimum, conditioned to its mean. It expresses an equity preference.
- Decreasing marginal utility (quasi-concavity or more general S–concavity)

Social Welfare and Inequality

If V is homogeneous of the 1st degree, then to derive the Gini index we have to separate inequality and mean effects. First, we write

$W = \mu V(\frac{x_1}{\mu}, \frac{x_2}{\mu}, ..., \frac{x_N}{\mu})$

If we normalize units as $V(1, 1, ..., 1) = 1$, when there is perfect equality, that is, everybody have individual level of welfare, social welfare has the same value. Note that we have that

$W = \mu(1 - I)$

By the transfers principle, inequality is the cost that makes the value of social welfare falls below the perfect equality point.

Now let's follow Atkinson's approach and use a SWF that is additive and such that

$W = \frac{1}{N} \sum_{i=1}^{N} \frac{x_i^{1-\varepsilon}}{1-\varepsilon}$, for $\varepsilon \neq 1$

Using

$I = 1 - [\frac{1}{N}\sum_{i-1}^{N}(\frac{x_i}{\mu})^{1-\varepsilon}]^{1-\varepsilon}$ for $\varepsilon \neq 1$,

we will have a inequality measure associated with this SWF. If we use a weight structure of individual incomes $w(x) = [1-F(x)]$, the inequality measure derived from the SWF will be the Gini index.

iv. As seen above, the social welfare indicator can be written as

$V = \mu(1 - I)$

which corresponds to a social welfare function defined over the income space. Note that I is not a usual measure of inequality such as the Gini index.

The idea of inclusive growth is developed as follows. First, take logs in the equation above:

$ln(V) = ln(\mu) + ln(1 - I)$

Taking the first difference, we have

$\gamma^* = \gamma + g$

where $\gamma^* = \triangle ln(\mu_s)$ is the growth rate of shared prosperity, $\gamma = \triangle ln(\mu)$ is the growth rate of average prosperity and $g = \triangle ln(1 - I)$ is the growth rate of equity in social welfare, which will be positive (negartive) if equity is increasing (decreasing). Thus, there will be a gain (loss) in the growth rate of social welfare when equity is improving (deteriorating).

For instance, if $\gamma^* = 6\%$ and $\gamma = 4\%$, it means that there is a gain of 2% in the growth rate of shared prosperity entirely attributed to the improvement of equity. This motivates the idea of inclusive growth, which can be measured by the gain in growth rate due to increasing so that the larger the gain, the greater is the social welfare growth. By the same token one may disaggregate in separate the determinants of mean and inequality growth into income sources (labor, social security, etc) and also do labor ingredients (earnings, occupation rate, etc).

B) i. Draw a sketch of the Lorenz Curve and calculate the Theil-T, the Gini and their duals using the follow income distribution: x = [1; 3; 4; 12]. ii. If we add one individual with null income in the sample, how do these 4 measures change?

Solution

i. We have that x = [1; 3; 4; 12]. The individual cumulative shares in total population are

$\pi = [\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1] = [0.25, 0.5, 0.75, 1]$

The individual cumulative shares in total income are

$s = [\frac{1}{20}, \frac{4}{20}, \frac{8}{20}, 1] = [0.05, 0.2, 0.4, 1]$

The Lorenz Curve is

We can calculate the Gini using the fact that it is one minus two times the area between the Lorenz Curve and the perfect unequality line. We have that this area is

$B = (\frac{0.25\times0.05}{2}) + [(0.25 \times 0.05) + (\frac{0.25\times0.15}{2})] + [(0.25 \times 0.2) + (\frac{0.25\times0.2}{2})] + [(0.25 \times 0.4) + (\frac{0.25\times0.6}{2})]$

$B = (0.00625) + [(0.0125) + (0.01875)] + [(0.05) + (0.025)] + [(0.1) + (0.075)]$
$\Rightarrow B = 0.2876$
Therefore, we have that
$G = 1 - 2B = 1 - 2 \times 0.2876 = 1 - 0.575 = 0.425$
Alternatively, we could also use the formula
$G = \frac{2}{n^2\mu} \sum_{i=1}^{n} ix_i - \left(1 + \frac{1}{n}\right)$
We have that $n = 4$ and $\mu = \frac{1+3+4+12}{4} = 5$. Therefore,
$G = \frac{2}{16\times5}[1 + 6 + 12 + 48] - \left(1 + \frac{1}{4}\right) = \frac{2}{80}(67) - \frac{5}{4} = \frac{134-100}{80} = \frac{34}{80}$
$\Rightarrow G = 0.425$
Rembember that the dual of the Gini is the Gini itself.
The formula for the Theil-T is
$T = \sum_{i=1}^{4} y_i \ln(ny_i) = \frac{1}{n\mu} \sum_{i=1}^{4} x_i \ln(\frac{x_i}{\mu})$
Therefore, we have that
$T = \frac{1}{20}[1 \times ln(\frac{1}{5}) + 3 \times ln(\frac{3}{5}) + 4 \times ln(\frac{4}{5}) + 12 \times ln(\frac{12}{5})]$
$\Rightarrow T \approx 0.324$
The dual of the Theil-T is
$U_T = 1 - \exp(-T) \approx 1 - 0.723 = 0.277$

ii. Now we have that x = [0;1; 3; 4; 12]. Adding an individual with zero income represents a share of $\phi = \frac{1}{5}$ with zero income in the new distribution. Using the fact that the dual of the Gini is the Gini itself, we have that if we add an individual with null income, the Gini of the new distribution will be

$G' = \phi + (1 - \phi) G = \frac{1}{5} + (\frac{4}{5}) \times 0.425 = 0.54$

The dual of the Theil-T for the new distribution is

$U'_T = \phi + (1 - \phi) U_T = \frac{1}{5} + (\frac{4}{5}) \times 0.277 = 0.422$

Therefore, the Theil-T for the new distribution is

$T' = -ln(1 - U'_T) \approx 0.547$

Note that we could also calculate the Gini drawing the Lorenz curve for the new distribution and doing the same procedure as in (i).

C) i. Calculate the Theil-T between groups using the following data: Men - Individual Income 600 and Population 100 million; Women - Individual Income 400 and Population 100 million. ii. Describe the limitation of gender related inequality measures such as these if based on per capita household income.

Solution i. Note that we are considering two representative individuals (one for the male and one for the female). Then, the within groups component will be equal to zero (only onde representative individual in each group). Therefore, we will have that the Theil-T index will be equal to the between groups component. Considering men as group 1 and women as group 2, we have that the shares in total population for each group are $\pi_1 = \frac{100M}{100M+100M} = \frac{1}{2} = 0.5$ and $\pi_2 = 0.5$.

The shares in total income are $Y_1 = \frac{600M}{(600M+400M)} = 0.6$ and $Y_2 = \frac{400M}{(600M+400M)} = 0.4$.

Therefore, we have that

$$T = T_b = \sum_{h=1}^{2} Y_h \ln \frac{Y_h}{\pi_h} = (0.6)\ln(\frac{0.6}{0.5}) + (0.4)\ln(\frac{0.4}{0.5}) \approx 0.02$$

ii. The limitation is that if we are based on per capita household income, we are implicitly assuming "socialization" within the household, that is, no inequality inside the household. Therefore, we could be underestimating actual gender inequality since we could have gender inequality inside the household.

D) i. Departing from the formula of the Sen's Poverty Index based on the Gini of the poor as the weight of the sum between P0 and P1 measures. Compare its advantages and disadvantages in relation to the poverty indicator known as the Mean Poverty Gap (P1). When the two are the same? Give the intuition. ii) Calculate the P1 of the Class of Poverty Indices of FGT and the minimum cost per person of the eliminating of poverty using the following sample and assuming a poverty line equal to 3: Rocinha period 1 = {1, 2, 6}; Rocinha period 2 = {2, 4, 6}.

Solution i.

We have that Sen's Poverty Index is given by

$PSen = P^0\delta^P + P^1(1-\delta^P)$

where δ^P is the Gini of the poor. Remember that

$P^1 = \frac{1}{N}\sum_{i=1}^{Q}(\frac{z-x_i}{z})$

P^1 has the property of taking into account the average cost of erradicating poverty, that is, the mean of the differences of incomes of the poor from the poverty line. The Sen's index is a convex combination of P^1 and P^0. P^0 is the less sensible index in terms of taking into account the severity of poverty. Therefore, if $\delta^P \epsilon (0,1)$, we have that the index is less sensible to measure poverty than P^1, which takes into account the distance of the poor individuals from the poverty line and not only the proportion of poor people (P^0). The two measures are equal when the Gini of the poor is equal to 0.

ii.

- Period 1

Note first that the proportion of poor people is $P^0 = \frac{2}{3}$ and the subsample of the poor is {1, 2}

The cost of erradicating misery is $(3-1)+(3-2) = 2+1 = 3$

It corresponds to $\frac{3}{3} = 100\%$ of the poverty line. Therefore, we have that the P_1^1 is equal to $\frac{3}{3} \times \frac{1}{3} = \frac{1}{3}$.

We could also calculate P^1 using the formula

$P_1^1 = \frac{1}{N}\sum_{i=1}^{Q}(\frac{z-x_i}{z}) = \frac{1}{3}\sum_{i=1}^{2}(\frac{3-x_i}{3})$

$P_1^1 = \frac{1}{3}\left[(\frac{3-1}{3}) + (\frac{3-2}{3})\right] = \frac{1}{3}(\frac{3}{3}) = \frac{1}{3}$

We have that the cost of erradicating poverty per individual is $\frac{3}{3} = 1$

- Period 2

Now the proportion of poor people is $P^0 = \frac{1}{3}$ and the subsample of the poor is $\{2\}$

The cost of erradicating misery is $(3-2) = 1$

It corresponds to $\frac{1}{3}$ of the poverty line. Therefore, we have that the P_2^1 is equal to $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

We could also calculate P^1 using the formula

$P_2^1 = \frac{1}{N} \sum_{i=1}^{Q} (\frac{z-x_i}{z}) = \frac{1}{3}(\frac{3-x_1}{3})$

$P_2^1 = \frac{1}{3}(\frac{3-2}{3}) = \frac{1}{3}(\frac{1}{3}) = \frac{1}{9}$

We have that the cost of erradicating poverty per individual is $\frac{1}{3}$

E) Using the same distributions of the item above (that is, Rocinha period 1 = {1, 2, 6} and Rocinha period 2 = {2, 4, 6}): i. Calculate the percentage of the P1 fall between these two periods associated with the income growth effect according to the Datt-Ravallion decomposition. ii) Check if there is first order dominance of the distribution in period 2 in relation to period 1 (consider the relevant range of poverty lines going up to 5). iii. What would this result imply in terms of the comparison of the Mean Squared Povety Gap (P2)?

Solution i. We have that $\mu_1 = 3$ and $\mu_2 = 4$, growth of 33.3%. Multiplying the first distribution, we have that

$(1 + 0.333)\{1, 2, 6\} = \{1.33, 2.66, 8\}$

We have that P1 for $\{1, 2, 6\}$ is

$P_1^1 = \frac{1}{3}\left[(\frac{3-1}{3}) + (\frac{3-2}{3})\right] = \frac{1}{3}(\frac{3}{3}) = \frac{1}{3} = \frac{3}{9}$

For $\{1.33, 2.66, 8\}$, we have that

$P_1^2 = \frac{1}{3}\left[(\frac{3-1.33}{3}) + (\frac{3-2.66}{3})\right] = \frac{1}{3}(\frac{1.67+0.34}{3}) = \frac{1}{3}(\frac{2.01}{3}) \approx \frac{2}{9}$

Therefore, we have that the fall of P1 associated with the income growth effect is

$\triangle P_1 = \frac{P_1^2 - P_1^1}{P_1^1} = \frac{\frac{2}{9} - \frac{3}{9}}{\frac{3}{9}} = -\frac{1}{3}$

That is, 33,3% of the fall in P1 is associated with the income growth effect.

ii. We have first order dominance. For every relevant range of the poverty line (going up to 5), we have that P0 for the distribution $\{1, 2, 6\}$ is always higher or equal to P0 for the distribution $\{1.33, 2.66, 8\}$.

iii. This implies that the Mean Squared Poverty Gap (P2) for the first distribution will be always higher than P2 for the second distribution, independent of the poverty line chosen. That is an implication of the theorem that says that $FOD \Rightarrow SOD \Rightarrow TOD$. That is, first order dominance implies second order dominance which in turn implies thir order dominance. Third order dominance, in turn, implies that P2 for the distribution that dominates the other one will be always higher, independent of the poverty line chosen.