Single slide to be provided in the exam

Inequality

Gini Index

- $\gamma = \frac{N+1}{N-1} - \frac{2}{N(N-1)\mu}\sum_{i=1}^{N}\rho_i x_i$
- $\gamma = \frac{1}{\mu N(N-1)}\sum_{i>j}^{N}\sum_{j}^{N}|x_i - x_j|$

Theil Measures

$$L = \sum_{i=1}^{n}\frac{1}{n}\log\frac{1/n}{y_i} = -\frac{1}{n}\sum_{i=1}^{n}\log\frac{y_i}{1/n}$$

$$T = \ln n - H(x) = \sum_i y_i \ln\frac{y_i}{1/n}$$

J-Divergence = T + L

J Decomposes variables **and categories**

$$J = \frac{1}{N\mu}\sum_{i=1}^{N}(x_i - \mu)\ln\left(\frac{x_i}{\mu}\right).$$

Variables Decomposition (for T, L & J)

$$T = T_e + \sum_{h=1}^{K} Y_h T_h$$

T = Te + Ti ; Te/T is the Contribution of a variable to inequality like in Mincer Regressions R^2 for Variance of Logs

General Entropy S- measure

$$S = \frac{1}{\varepsilon(1-\varepsilon)}\left[1 - \frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i}{\mu}\right)^{1-\varepsilon}\right]$$

ε=0 Theil T; ε=1 Theil L;

DUAL - A dual distribution follows :

$U_2 = \phi + (1-\phi)U_1$ Theil –T Dual: $T2 = T1 - \ln(1-\phi)$

The Dual of the Gini Index is the Gini Index

Derivation from Social Welfare Function

Atkinson Index for $\epsilon \neq 1$

- $W = \frac{1}{N}\sum_{i=1}^{N}\frac{X_i^{1-\varepsilon}}{1-\varepsilon}, \varepsilon \neq 1$

Inequality trough the Atkison Index

- $I = 1 - [\frac{1}{N}\sum_{i=1}^{N}(\frac{Xi}{\mu})^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}}$

Gini

$$W = \mu(x*) = \int_0^{\infty} u(x)w(x)f(x)dx$$

If u(x) = x and w(x) =2 [1 – F(x)]

$$W = \mu(1 - G)$$

Dynamic Decomposition:

$$Ln(W\ ) = Ln(\mu) + Ln(1-G)$$

$$\gamma^* = \gamma + g$$

$$\gamma^* = \Delta Ln(W\ ) \text{ etc}$$

Shared Prosperity

$$\mu_s = \mu(1 - I)$$

$$I = 1 - \frac{\mu_s}{\mu}$$

Dynamic Decomposition by Income Source:

$$\Delta Ln(\mu_{st}) \sim \frac{1}{2}\sum_{i=1}^{k}(\frac{\mu_{is(t-1)}}{\mu_{s(t-1)}} + \frac{\mu_{ist}}{\mu_{st}})\Delta Ln(\mu_{it})$$

Inequality of Opportunity

$$I_o = 1 - \frac{\vartheta_s}{\vartheta} \quad ; -1 \leq I_o \leq 1$$

$$\vartheta_s = \vartheta(1 - I_o)$$

Poverty

FGT Indicator

- $P^{\alpha} = \frac{1}{n}\sum_{i=1}^{q}(\frac{Z-Yi}{Z})^{\alpha}$

Poverty Index

Sen

- $P_s = P^0\delta + P_1(1-\delta^P)$

Watts

- $P_W = (\frac{1}{N}\sum Ln(\frac{Z}{y_i})$

Clark, Hemming and Hulp (1981)

- $P_{C-H-U} = (\frac{1}{nc})\sum[1 - (\frac{y_i}{Z})^c]$

Multidimensional Poverty

$MPI = H \cdot A \qquad H = \frac{q}{n} \qquad A = \frac{\sum_i^q c_i}{q}$

Global Social Indicators

Human Development Index (HDI)

- $HDI = \sqrt[3]{IhXIeXIi}$

where:
lh = health index;
Ie = education index;
Ii = income index

Inequality-adjusted HDI (IHDI)

- $Ax = 1 - \frac{\sqrt[g]{X_1 \ldots X_n}}{X}$
- $Ix^* = (1 - Ax)Ix$
- $IHDI = \sqrt[3]{Ih^* * Ie^* * Ii^*}$

Polarization (Alienation & Identification)

$$W_B = \int_0^{\infty} u(x)v(x)f(x)dx = \mu - (m_2 - m_1) + 2\mu G$$

The relative loss of social welfare due to Polarization

$$B = 2(G_B - G_W)$$