

Growth and Shared Prosperity in Brazil

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References:

* 9 these slides

https://www.cps.fgv.br/cps/bd/curso/9PP_Neri_SharedProsperity_MDShort.pdf

*10 Paper

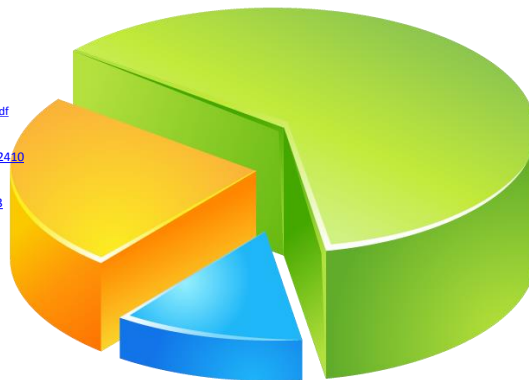
<http://www.cps.fgv.br/cps/bd/curso/10-Growth-and-Shared-Prosperity-in-Brazil.pdf>

*** Video Paper Presentation Short 4 min (Port)

<http://cps.fgv.br/videos/anpec-growth-and-shared-prosperity-brazil-integra-2410>

*** Video Paper Presentation Long 24 min (Port)

<http://cps.fgv.br/videos/anpec-growth-and-shared-prosperity-brazil-0353>



1

A Simple Indicator of Shared Prosperity = the 10th SDG

Following Atkinson (1970), we can write a general social welfare function denoted as:

$$W = \mu(x^*) = \int_0^{\infty} u(x)w(x)f(x)dx$$

Suppose x is the income of an individual which is a random variable with density function $f(x)$, then the mean income of the population is defined as:

$$\mu = \int_0^{\infty} xf(x)dx \quad (1)$$

The idea of shared prosperity (SP) is simply the mean of the bottom 40% of the population. More formally, suppose z is the income defined by:

$$0.4 = \int_0^z f(x)dx \quad (2)$$

then the Shared Prosperity (SP) indicator is defined by:

$$\mu_s = \frac{\int_0^z xf(x)dx}{\int_0^z f(x)dx} \quad \text{This should satisfy: } \int_0^{\infty} w(x)f(x)dx = 1$$

which shows that the Shared Prosperity (SP) indicator is a weighted average of individual incomes. New SDGs focuses on the bottom 40% of the population. The idea is that a large proportion of the population should take part in and benefit from the growth process.

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A Simple Indicator of Shared Prosperity

Applying the same decomposition the SP welfare indicator can be written as:

$$\mu_s = \mu(1 - I) \quad (3)$$

Where like other measures of social welfare defined over individual incomes such as Atkinson's (1970), it has an implicit (relative) inequality measure, defined as:

$$I = 1 - \frac{\mu_s}{\mu} \quad (4)$$

which is a similar form of social welfare functions defined over the income space such as Atkinson's and Sen's indexes. Note that I is not a usual measure of inequality such as the Gini index because it does not satisfy the weak transfer axiom. It will be referred to as a measure of inequity in shared prosperity and $(1-I)$ as a measure of equity in shared prosperity.

Or an Absolute measure of inequality:

$$A = \mu - \mu_s \quad (5)$$

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Prosperity and Inequality in Brazil

2001 - 2013

Average and Shared Prosperity and Inequality in Brazil

Year	Average prosperity	Shared prosperity	Absolute inequality	Relative Inequality (%)
2001	7717	1581	6136	79.52
2002	7725	1644	6081	78.72
2003	7272	1577	5695	78.31
2004	7514	1736	5778	76.89
2005	7976	1870	6106	76.55
2006	8724	2113	6611	75.78
2007	8945	2198	6747	75.43
2008	9373	2405	6968	74.34
2009	9630	2481	7149	74.24
2011	10235	2791	7444	72.73
2012	11020	3068	7952	72.16
2013	11405	3169	8237	72.22
Trend 2001-2013	341.11	142.02	199.09	-0.63

Source: Author's calculation from PNAD/IBGE microdata

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Shared Growth

$$\mu_s = \mu(1 - I) \quad (3)$$

The idea of shared growth is now developed. To do so write (3) as:

$$\ln(\mu_s) = \ln(\mu) + \ln(1 - I) \quad (5)$$

which on taking the first difference gives:

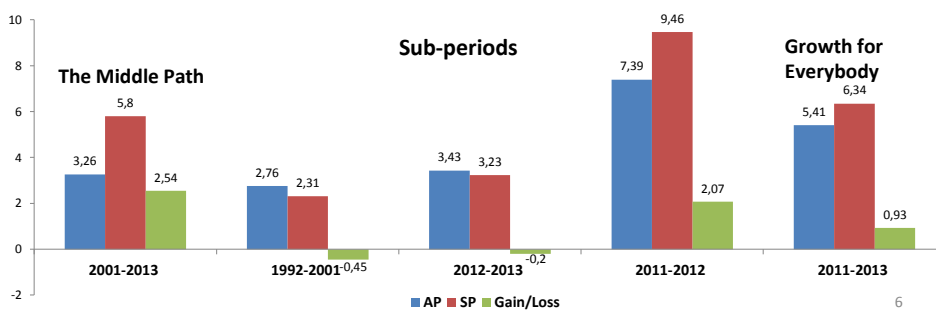
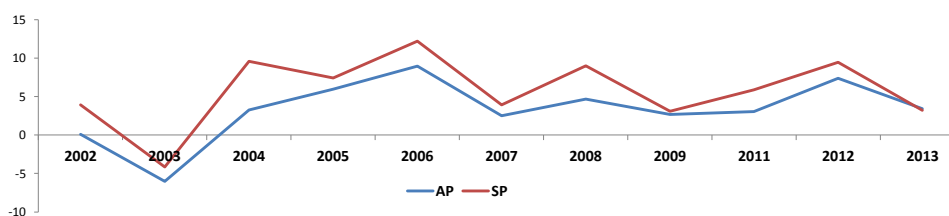
$$\gamma^* = \gamma + g \quad (6)$$

where the first term is the growth rate of shared prosperity, the second term is the growth rate of average prosperity and g is the growth rate of equity in shared prosperity, which will be positive (negative) if equity in shared prosperity is increasing (decreasing). Thus, there will be a gain (loss) in growth rate when equity is improving (deteriorating).

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Average and Shared Prosperity

Annual Growth Rates of Average (AP) and Shared Prosperity (SP)– percentage points (p.p.)



Source: Author's calculation from PNAD/IBGE microdata

6

Income Sources as Determinants of Shared Growth Trends in Brazil

Suppose μ_t is the AP in year t and μ_{it} is the mean of the i th income component in year t .

$$\mu_t = \sum_{i=1}^k \mu_{it} \quad (15)$$

Then it can be showed that:

$$\Delta \ln(\mu_t) \sim \frac{1}{2} \sum_{i=1}^k \left(\frac{\mu_{it}(t-1)}{\mu_{it}(t-1)} + \frac{\mu_{it}}{\mu_t} \right) \Delta \ln(\mu_{it}) \quad (16)$$

which shows that the growth rate of AP is the weighted average of the growth rates of individual income components - the weights being proportional to the average of income shares in each period. This equation informs the magnitude of contribution of each income component to the growth rate of AP. Similarly, we can explain the contribution of each income component to growth rate of SP using:

$$\Delta \ln(\mu_{st}) \sim \frac{1}{2} \sum_{i=1}^k \left(\frac{\mu_{ist}(t-1)}{\mu_{ist}(t-1)} + \frac{\mu_{ist}}{\mu_{st}} \right) \Delta \ln(\mu_{it}) \quad (17)$$

where μ_{st} is the SP in year t and μ_{it} is the mean of the i th component of the bottom 40% of the population in year t . This equation informs the magnitude of the contribution of each income component to the growth rate of SP. Shared growth is defined as the gain/loss in the growth rate of the SP, which is the difference between the growth rates of SP and AP. The difference of growth in (16) from (17) provides the contributions of each income component to shared growth.

Which source of income contributed the most to growth in total income?

Making a long story short:
Lego type decomposition
of growth rates
2001 - 2013

Contribution of Income Sources to Growth by Different Income Groups in

	AP Mean	10% Poorest	SP - 40% Poorest	Middle Group 40%-90%	5% Richest
Labor	2.47	2.63	3.86	2.78	1.82
Non-Labor	0.79	3.74	1.93	1.05	0.19
Total	3.26	6.37	5.80	3.82	2.01

At this growth pace, it would take **34,8** years at this rate for the 5% richest to double its income against **12,3** years for the 40% poorest.

Double

$$X_{t2} = 2 \cdot X_{t1} = (1+r)^T \cdot X_{t1}$$

Taking logs: $T = \ln 2 / \ln (1+r)$

Question: What would be the annual rate that leads to a poverty fall of 50% in 25 years?

Which non labor source of income contributed the most to growth in total income? 2001 – 2013

Contribution of Income Sources to Growth by Different Income Groups in annualized p.p.

2001-2013	Mean	10% Poorest	40% Poorest	Middle Group 40-90	5% Richest
Labor	2.47	2.63	3.86	2.78	1.82
BPC	0.09	0.19	0.28	0.12	0.00
S. Security	0.68	0.34	0.88	0.95	0.28
Other (including BF*)	0.07	3.42	0.82	0.03	-0.05
Total	3.26	6.37	5.80	3.82	2.01

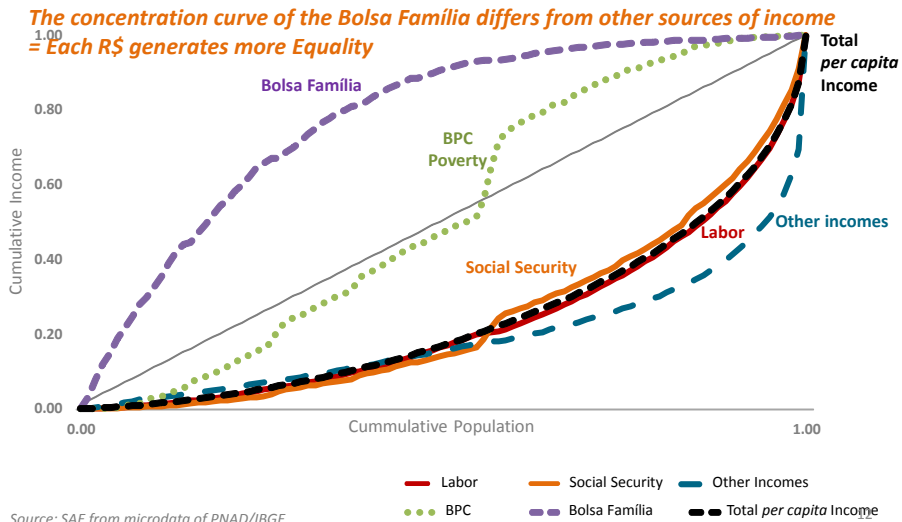
2001-2012	Mean	10% - Poorest	40% - Poorest	5% +
Bolsa Família	0.10	3.29	0.83	0.00

Source: SAE from PNAD/IBGE microdata

Pro-Shared Prosperity Index (Targeting Efficiency) and Concentration Curves Different Sources 2012

$$\varphi_i = \frac{\mu_{is}\mu}{\mu_s\mu_i}$$

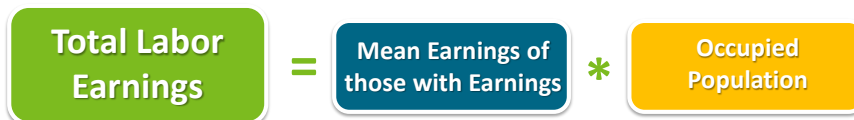
Year	Labor	BFP	BPC	S-security	Other	Total
PSPI - 2012	0.94	7.09	3.25	0.84	0.88	1.00



Source: SAE from microdata of PNAD/IBGE

Labor Deconstruction

Per Capita Labor Income in the total population can be expressed as:



We can continue decomposing each piece of the identity in elements, what helps to understand the relative weight of each labor ingredient.



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Which labor ingredient contributed the most to growth in labor income?

2001 - 2013

Contribution of Income Sources to Growth in Labor Income in annualized p.p

	Mean	10% Poorest	40% Poorest	Middle Group	5% Richest
Employment Rate	0.25	-0.06	0.11	0.32	0.11
Participation Rate	0.38	-0.90	-0.19	0.61	0.79
Hours Worked	-0.39	-0.75	-0.44	-0.43	-0.21
Hourly Wages	2.93	5.59	5.60	3.07	1.64
Labor	3.18	3.88	5.08	3.57	2.33

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Source: SAE from PNAD/IBGE microdata

Which labor ingredient contributed the most to growth in labor income?

2001 - 2013

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Employment Rate	0.25	-0.06	0.11	0.32	0.11
Participation Rate	0.38	-0.90	-0.19	0.61	0.79
Hours Worked	-0.39	-0.75	-0.44	-0.43	-0.21
Years of Schooling	2.23	4.90	4.22	1.97	0.54
Premium per School Year	0.69	0.70	1.38	1.10	1.10
Labor	3.18	3.88	5.08	3.57	2.33

Source: SAE from PNAD/IBGE microdata

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Shared Opportunities

2001 - 2013

Trends in Average and Shared Opportunities in 2001-2013

Type of Opportunity	Average Opportunity	Shared Opportunity
Occupation Opportunities (Employment Rate)	0.28	0.15
Productive Employment (% with formal contract)	1.00	1.45
Education Attainment (Years of Schooling)	0.17	0.21
School Attendance (% 6-14 years old attending school)	0.26	0.39

Source: Author's calculation from PNAD/IBGE microdata

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Shared Opportunities

Suppose $o(x)$ is an opportunity enjoyed by a person with income x , then the average opportunity (AO) enjoyed by the society is given by:

$$\vartheta = \int_0^{\infty} o(x)f(x)dx$$

This is the average opportunity available, but does not inform how it is shared by the population. Similar to the idea of shared prosperity we can define shared opportunity (SO) as:

$$\vartheta_s = \frac{\int_0^z o(x)f(x)dx}{\int_0^z f(x)dx}$$

which is the average opportunity enjoyed by the bottom 40% of the population. The inequity in opportunity can be defined as:

$$I_o = 1 - \frac{\vartheta_s}{\vartheta}$$

Then the SO can be written as:

$$\vartheta_s = \vartheta(1 - I_o)$$

which is similar to Atkinson's and Sen's social welfare functions but defined over opportunity space. I_o measures the proportional loss (or gain) in opportunity due to inequity (or equity) and therefore can be an indicator of inequity (equity) in opportunity. Note that unlike inequity measure I defined in (3), which lies in the range $0 \leq I \leq 1$, this inequity measure I_o lies in the range $-1 \leq I_o \leq 1$. The negative (positive) value implies that that opportunity is inequitable (equitable).

Occupation Opportunities

2001 - 2013

Average and Shared Opportunity in Employment Rate - %

Year	Average Opportunity	Shared Opportunity	Inequity
2001	90.50	85.18	5.88
2002	90.73	85.67	5.57
2003	90.16	84.11	6.71
2004	90.88	85.26	6.18
2005	90.47	84.86	6.20
2006	91.39	85.63	6.31
2007	91.67	85.80	6.41
2008	92.70	87.36	5.76
2009	91.52	84.86	7.28
2011	93.08	86.42	7.15
2012	93.62	87.09	6.98
2013	93.30	86.30	7.51
Trend 2001-2013	0.28	0.15	0.12

Source: Author's calculation from PNAD/IBGE microdata

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