Income Distribution & Inequality

Ginis, (Lorenz 01.03) & Graphical Representation World and Brazil – 01.071 and 01.072:

Various optional materials from 01.06 to 01.0722

OBS: The most relevant material normally used in class will be posted directly in the https://cps.fgv.br/en/courses-short opening page. While longer and more detailed material will be included within blocks directed by these numbers sequence above :

Marcelo Neri
FGV Social

GLOBAL INCOME DISTRIBUTION PORTRAIT – WHERE ARE BRAZIL, RELATIVE TO THE BRICs,THE US & THE WORLD

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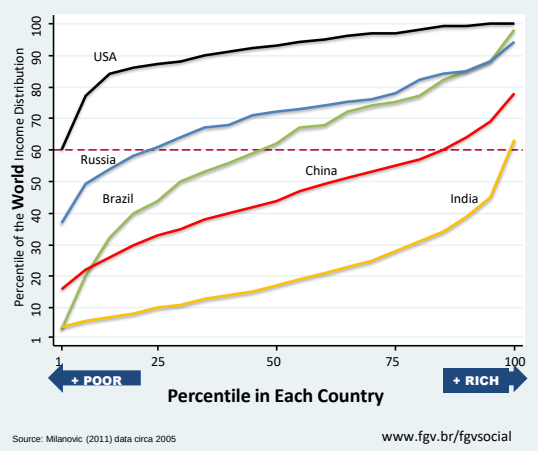
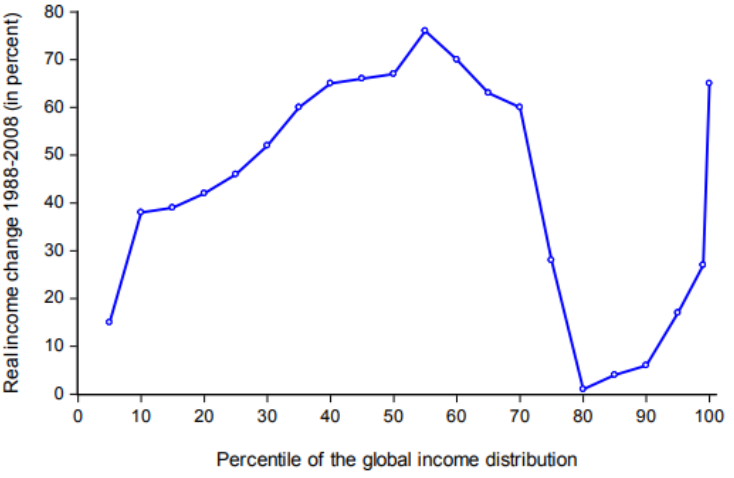


The Elephant Graph - Change in real income between 1988 and 2008 at various percentiles of global income distribution (2005 international dollars)

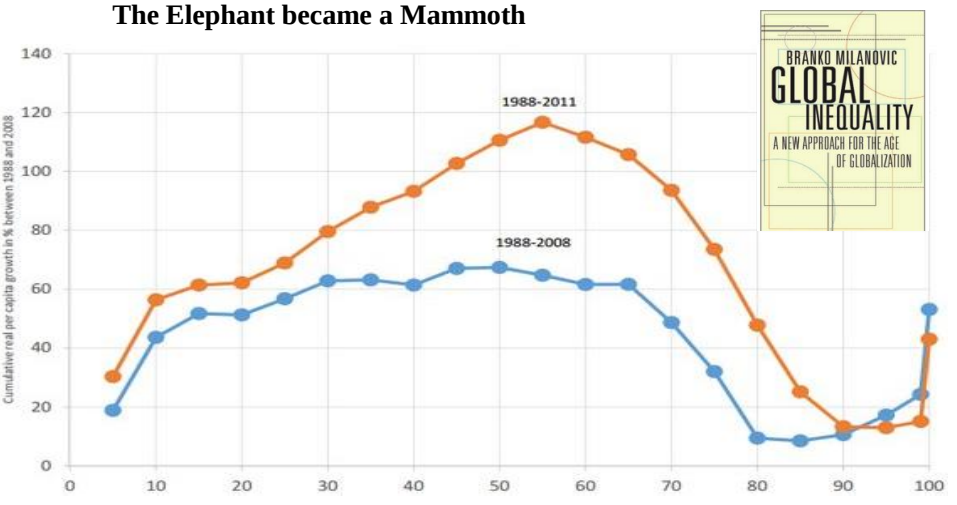


Source: Ravallion (2018) based on Milanovic (2016)

Definition: **The Growth Incidence Curve (GIC)** refers to the growth rate of income for every quantile (percentile, decile etc) of the income distribution between 2 points in time.

Impressive cumulative change just in 3 years – Decoupling of the BRICS with respect to Developed countries at the on set of the 2008 Global crisis

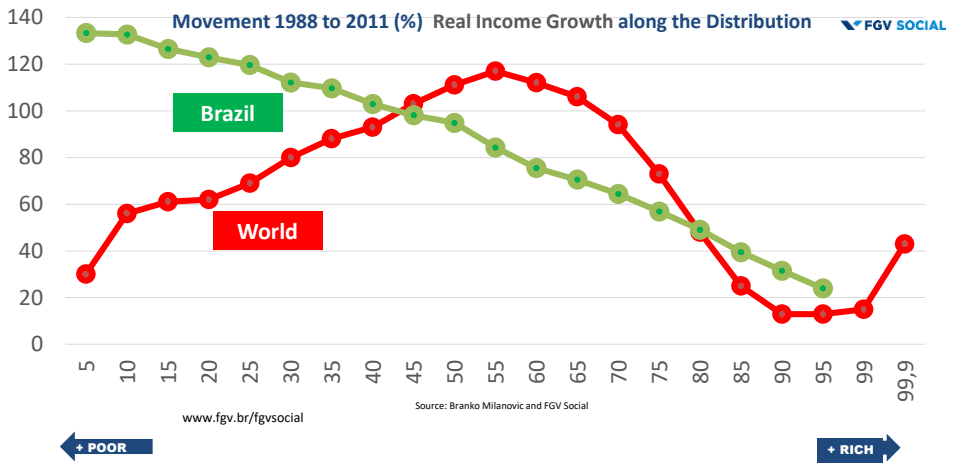
The Elephant became a Mammoth



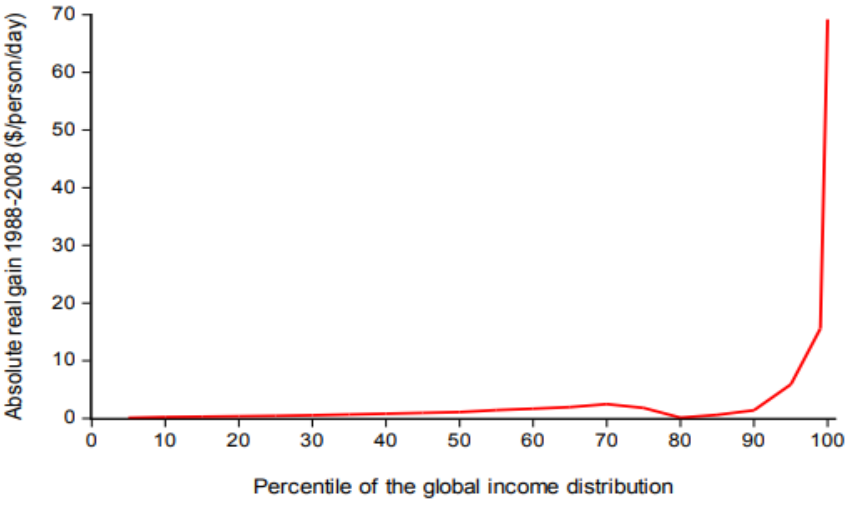
Source: Milanovic (2016), Chapter 1

According to this graph there is an improvement to every part of the distribution - Pareto Improvement under <u>Anonymity</u>

How about Brasil in this period?

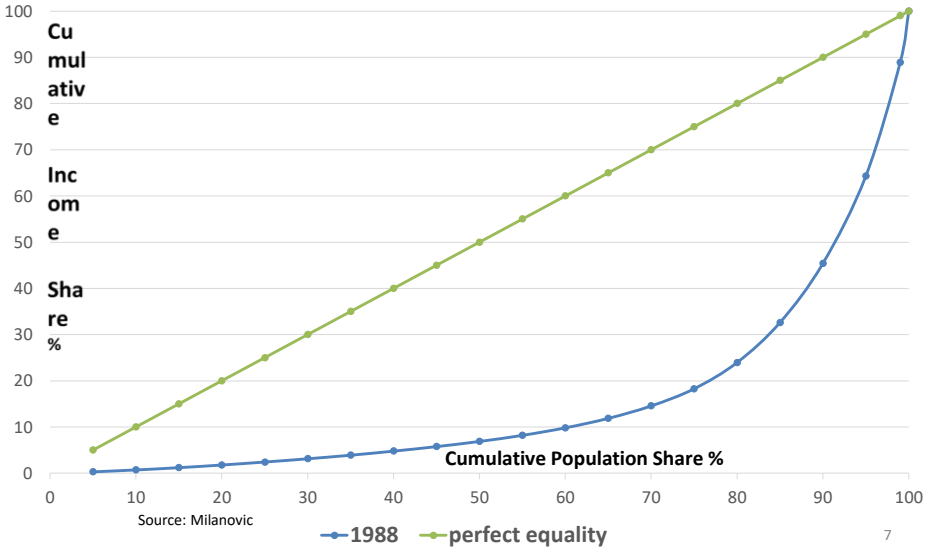


The Serpent graph: absolute real gain 1988-2008
($/person/day)



Source: Ravallion (2018)

Lorenz curve 1988 - Global



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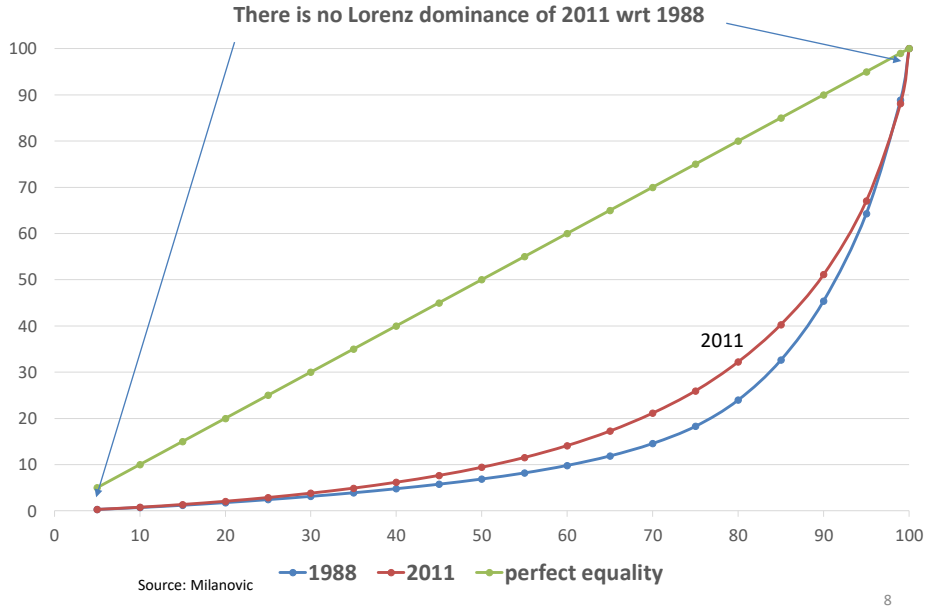
Lorenz curves 1988 and 2011 - Global



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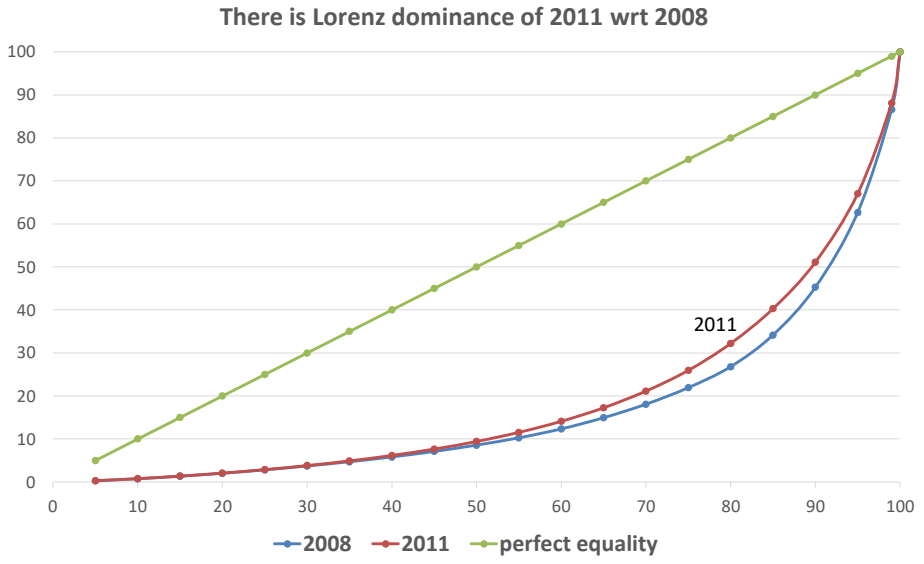
Lorenz curves 2008 and 2011 - Global



Source: Milanovic

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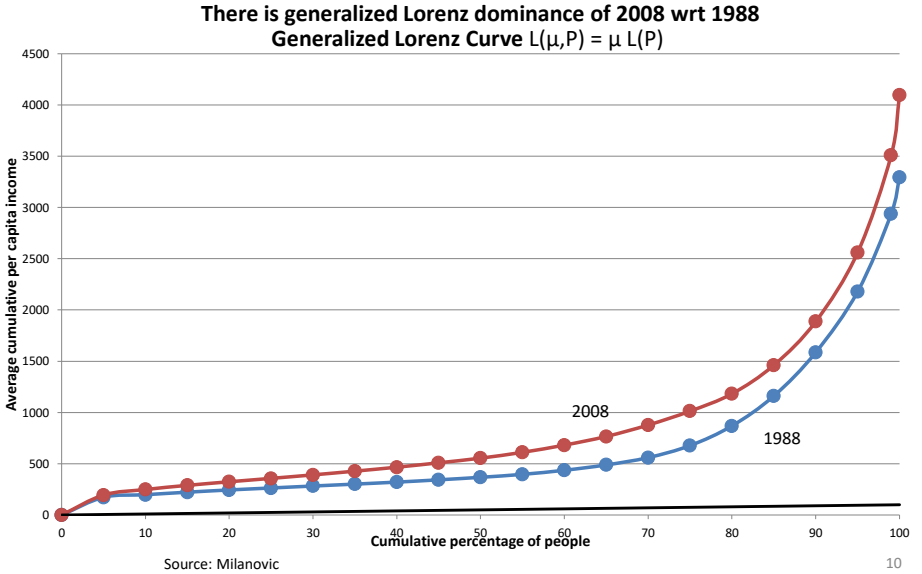
Generalized Lorenz curves 1988 and 2008 - Global



Source: Milanovic

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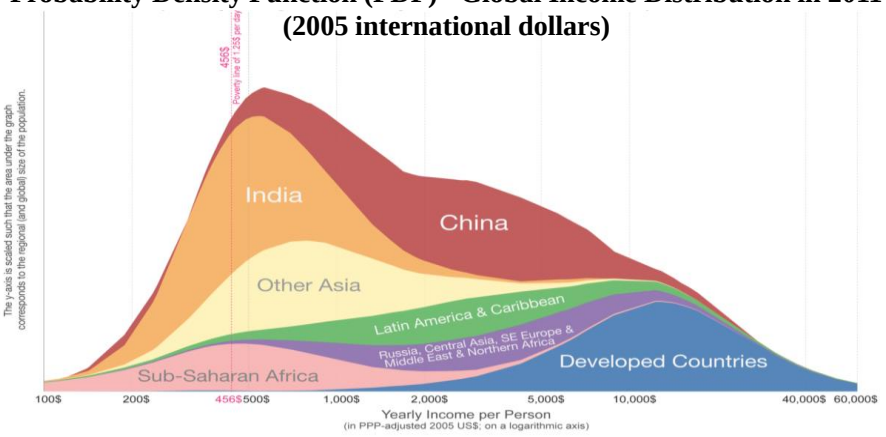
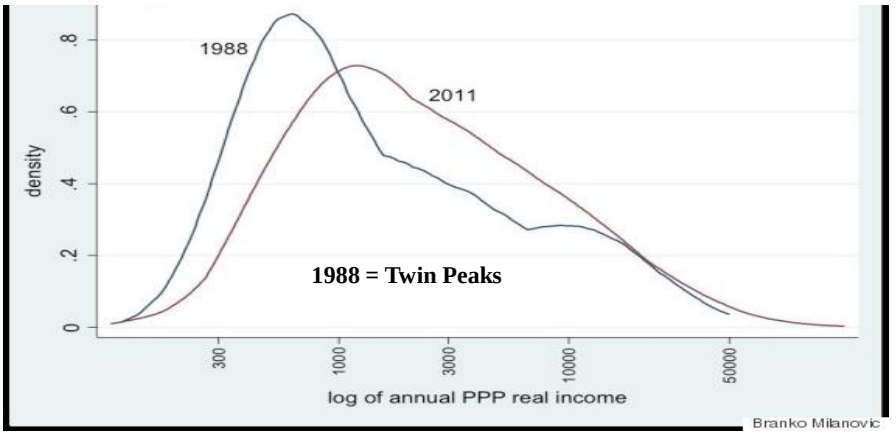
Probability Density Function (PDF) - Global Income Distribution in 2011 (2005 international dollars)



Source: Lakner and Milanovic 2015

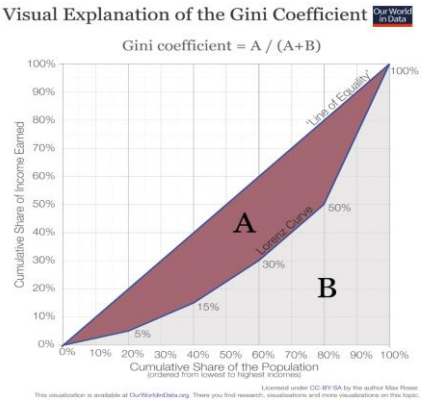
Global income distribution in 1988 and 2011 (levels) – Probability Density Function (PDF)



Source: Milanovic (2016), Chapter 1

Gini Index *01.02

Graph from the Lorenz Curve L(P)



Analytical Interpretation

Once there are $\frac{N\,(N-1)}{2}$ distinct pairs of people in the sample, Gini's formula is:

$$\gamma = \frac{1}{\mu N(N-1)} \sum_{i>j}^{N} \sum_{j}^{N} |x_i - x_j|$$

Alternative Formula

$$\gamma = \frac{N+1}{N-1} - \frac{2}{N\,(N-1)\mu} \sum_{i=1}^{N} \rho_i\, x_i$$

Aggregated Level Analysis of Welfare Function (BES) based on the Gini (δ)

Sen (1976): $\mu\,(1 - \delta)$

Graff (1981): $\mu\,(1 - \delta)^{\rho}$, where $\rho \in [0, 1]$.

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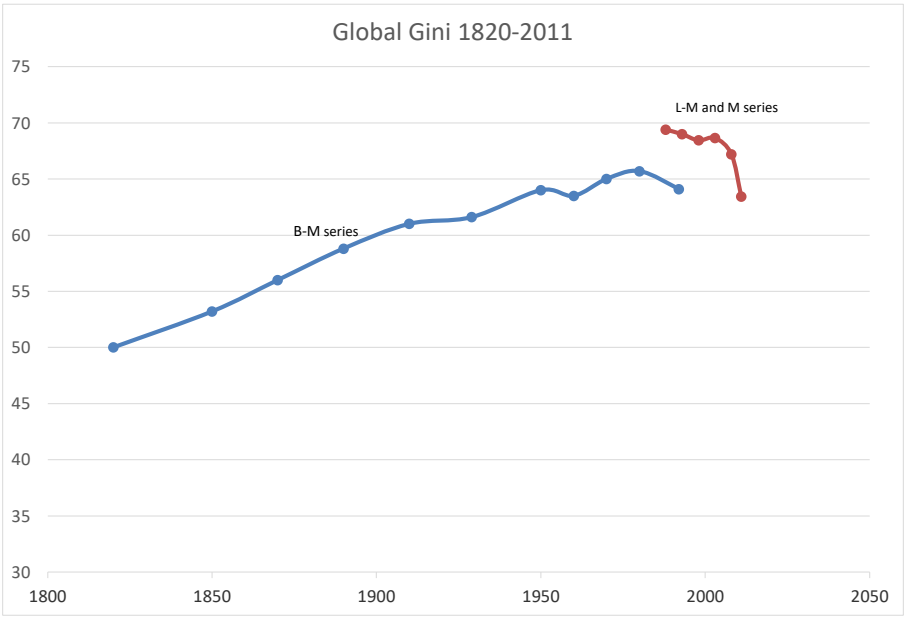
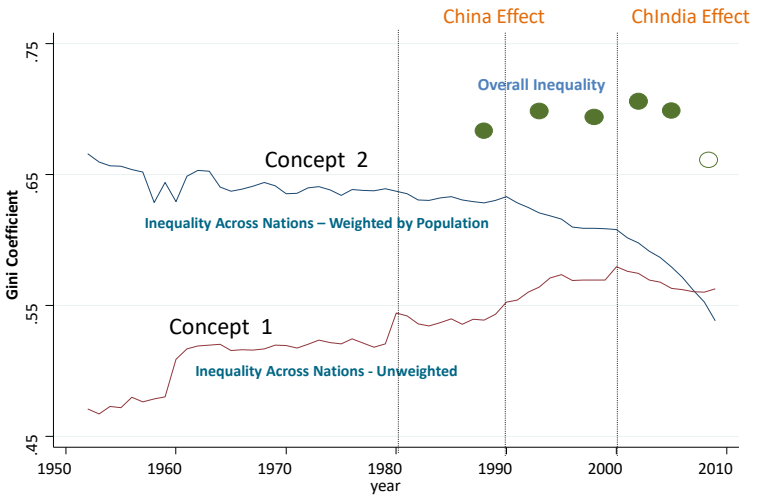


Branko Milanovic (2016)

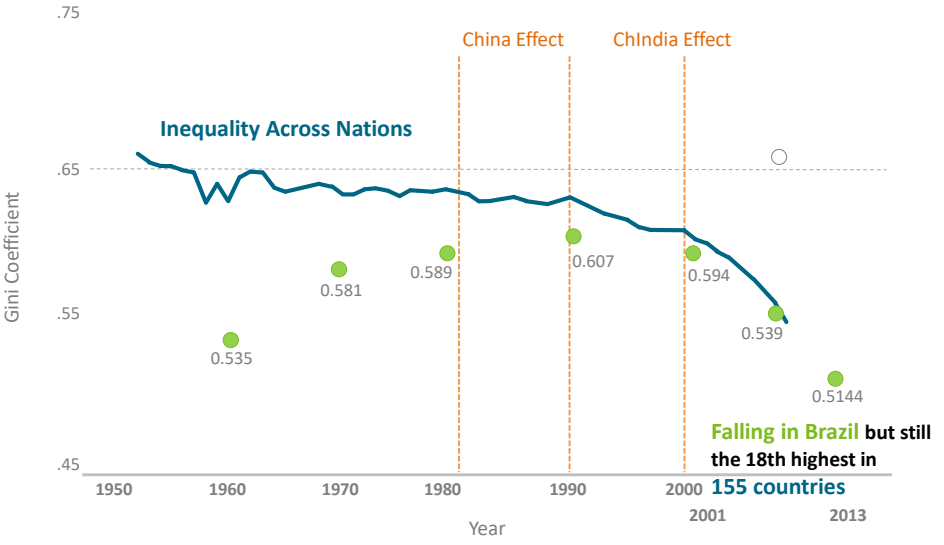
Evolution of World Income Inequality Different Concepts



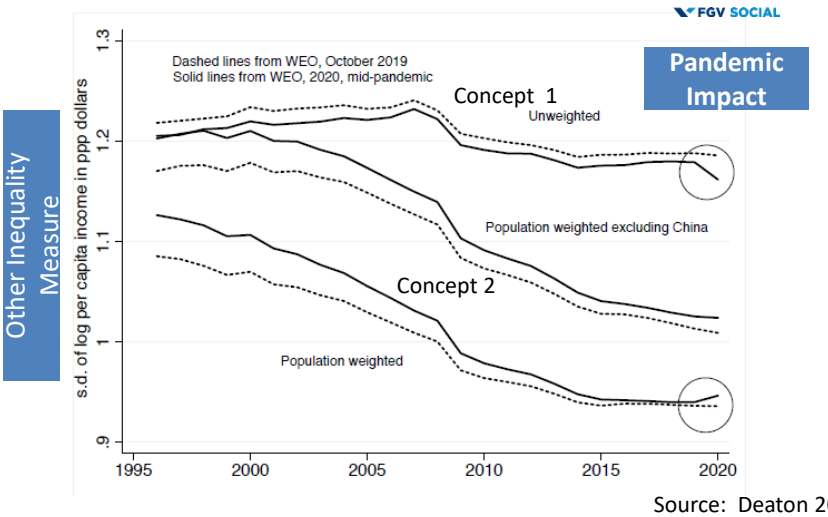
Source: Milanovic (2011)

Brazil is a small World

Brazilian Per capita GDP PPP was 94% of Global Levels in 2012. It has grown 3.5% agains 3.6% of the world between 2002 e 2012



Source: Milanovic (2011) and Neri (2011)



Source: Deaton 2021

[Deaton's (2021)Paper and Video debate with Deaton and Milanovic on pandemic impact on World Inequality (between countries) :]

** https://www.nber.org/papers/w28392

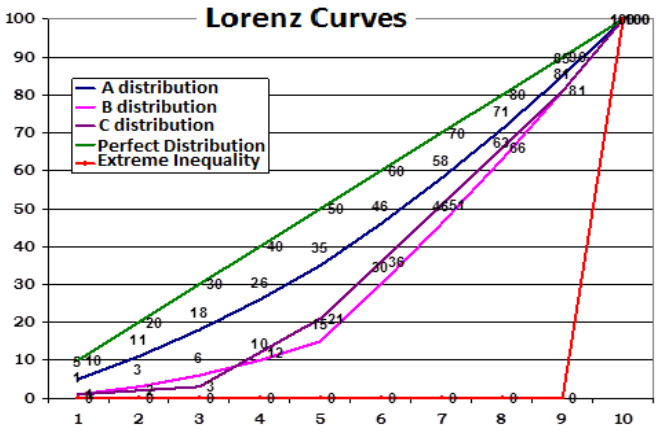
** https://www.facebook.com/ebrdhq/videos/238841154356288

LORENZ CURVE is a simple graphic instrumental that allows the description of the income distribution in a given society, besides helping the ordering of different distributions departing from a welfare point of view (Max Otto Lorenz 1905). Lorenz curve express the relation between the cumulative proportion of people with income at least equal to some specific value and the cumulative proportion of income received by these people. Lorenz curve is represented by a function L(P), which corresponds to the a fraction received by the p-th lower fraction of the population, when it is ordered by increasing income. The curve slope is always positive and convex, so L(0) = 0 and L(1) = 1.

Lorenz Dominance - We say that the Lorenz curve of a distribution A dominates distribution of B if curve A is above curve B in all points of the distribution. In this case the one can say A is more equal than B. And if both distributions have the same mean, A is preferable to B.

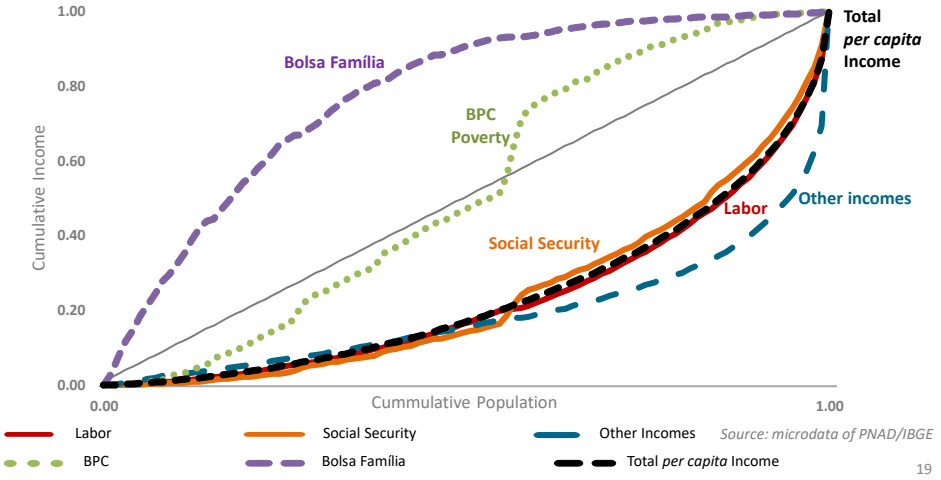
Lorenz 01.03

	A Distribution		B Distribution		C Distribution		Lowest Inequality		Highest Inequality	
Individuals		Accumulated		Accumulated		Accumulated		Accumulated		Accumulated
1	5	5	1	1	1	1	10	10	0	0
2	6	11	2	3	1	2	10	20	0	0
3	7	18	3	6	1	3	10	30	0	0
4	8	26	4	10	9	12	10	40	0	0
5	9	35	5	15	9	21	10	50	0	0
6	11	46	15	30	15	36	10	60	0	0
7	12	58	16	46	15	51	10	70	0	0
8	13	71	17	63	15	66	10	80	0	0
9	14	85	18	81	15	81	10	90	0	0
10	15	100	19	100	19	100	10	100	100	100



Concentration Curve - The Concentration Curves are a representation that bears similarities to the Lorenz Curve. However, while the latter refers to the distribution of a single variable throughout the population, the former are constructed from the distribution of two variables in the population In fact, the Lorenz Curve can be understood as a particular case of the Concentration Curve where the variable used in the ordering of the population and the output variable coincides. Similarly, the correspondence between Gini Index and the Lorenz Curve also appears in the relationship between the Concentration Curve and the Concentration Index. The difference is that the Gini varies between 0 and 1 while the Concentration Index varies between -1 and 1. If a certain attribute is more directed to the poor, for example conditional cash transfers, then the indicator is negative. See examples in the graphs below.

The concentration curve of different sources of income = Each R$ generates more Equality



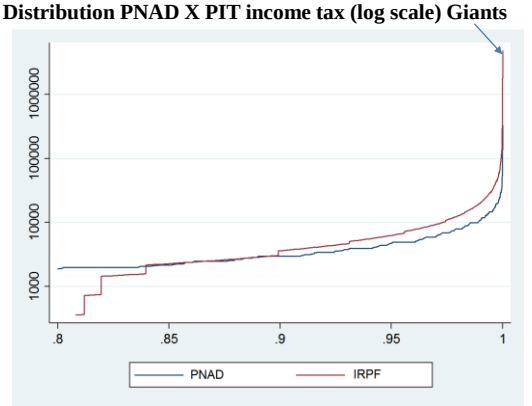
Source: microdata of PNAD/IBGE

Generalized Lorenz Curve - Lorenz curves in imply welfare dominance only when one compares distributions with the same mean, a rather restrictive hypothesis. Shorrocks (1983) and Kakwani (1984) developed a criterion to compare distributions with different mean. The Generalized Lorenz Curve is a modification of the Lorenz Curve in which the accumulated fraction of incomes up to each fraction of the population is multiplied by the average income of the distribution. Because of this multiplication, the generalized curve brings information about the form and level of the distribution, or the joint first two moments of distribution, such as the income distribution curve and its congeners of basic statistics. Lorenz Generalized Curve is represented by a function $L(\mu,P) = \mu L(P)$.

If the Lorenz Generalized Curve of the distribution of A is above the curve of the distribution of B in all points, then the social welfare associated with A will be unequivocally superior then the welfare associated with B, for any symmetric welfare function (satisfy anonymity property) and quasi-concave (i.e. it satisfies Pigou Dalton property).

Pen's Parade (Pen 1971) is a metaphor used to describe an income distribution. In it, income inequality is associated with inequality in people's height. This feature draws attention to the fact that if the height of people were proportional to their incomes, we would live in a society formed by a large mass of dwarves and a small elite of giants. There are other graphical representations of this distribution, usually a little different from the image of a crowd of dwarves. Imagine a population ordered by income, with the poorest people first and the richest afterwards. If we divide this population into one hundred equal parts, we will have one hundred percentiles, or twenty vintiles, and so on, that we refer here generally to the **income distribution curve**. That is nothing more than the representation of a cumulative distribution function (CDF) of incomes, with the axes reversed. Following the basic analogy with basic statistics there is still the density curve of income distribution.

Pen's Parade: Truncated Top Part of Adults Income Distribution PNAD X PIT income tax (log scale) Giants



Source: IRPF means PIT (Personal Income Tax) – Brazil 2014.

Distribution of Income : Graphic Approach – Levels and Changes