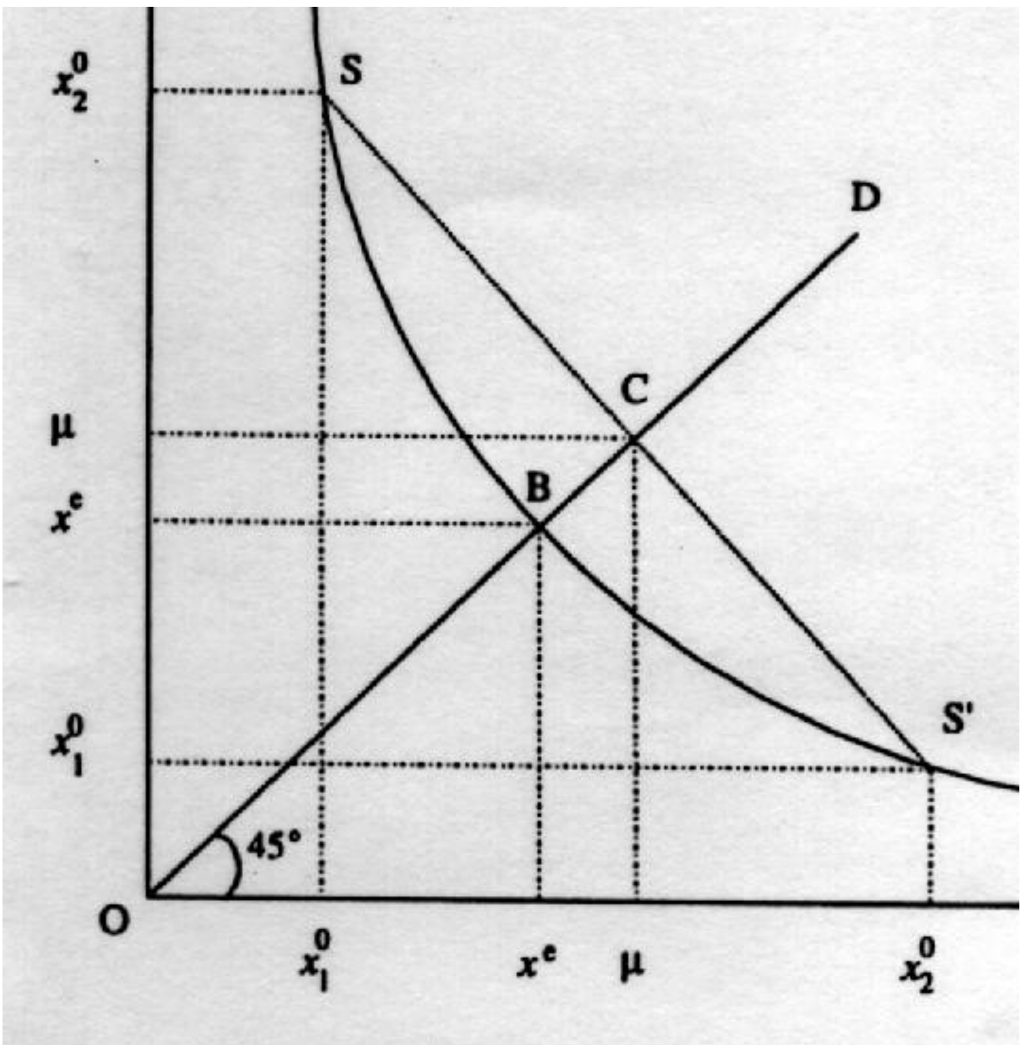
SOCIAL WELFARE *01.04

References:
*01.05 Deaton (1997), Chapter 3, section 3.1 (section 3.2 will be used on the Poverty part)
₴ Based on Atkinson’s classic “On The Measurement of Inequality” (1970)

Social Welfare Function (SWF)

$$W = V(x_1, x_2, \ldots, x_N)$$



SWF function is a sum across individuals (typically of per capita expenditures or income) .

Properties of BES Functions

- Pareto Optimum – V is increasing (non decreasing) in its arguments. If one gets better and nobody worse it increases – to accommodate poverty measures (truncated BES functions) we adopt non decreasing function.
- Symmetry or Anonymity – BES depends on individual welfare levels and not their identity.
- Principle of Transfers (Pigou – Dalton) – For a given total X, BES function will be at it maximum point when inequality will be at the same time at its minimum, conditioned to the average (when OC' S are equal) – express an equity preference. Ignore any kind of restrictions on allocations and incentives effects.

  Decreasing marginal utility (quasi-concavity or more general S – concavity) . If $\mathbf{x_1}$ and $\mathbf{x_2}$ are lists of x's and if V ($\mathbf{x_1}$) =V ($\mathbf{x_2}$) then $\lambda \mathbf{x_1} + (1 - \lambda) \mathbf{x_2}$ for a λ – [0 , 1] will have a higher value or equal to the original allocations.

SOCIAL WELFARE AND INEQUALITY

If V is homogeneous of the 1st degree

$$W = \mu V\left(\frac{x_1}{\mu}, \ldots, \frac{x_N}{\mu}\right)$$

Separate inequality and mean effects

If we normalize units as V (1,1,...,1) = 1

When there is perfect equality, that is, everybody have individual level of welfare, social welfare has the same value.

$$W = \mu (1 - I)$$

By the transfers principle, inequality is the cost that makes the value of social welfare falls below the perfect equality point.

X^e is the equivalent of x equally distributed

BC/OC is the geometric measure of inequality proposed by Atkinson.

One advantage of this approach is to differentiate inequality and social welfare. It is consistent with poverty.

From: Social Welfare Function **To: Inequality**

$$W = \frac{1}{N}\sum_{i=1}^{N}\frac{x_i^{1-\epsilon}}{1-\epsilon}, \quad \epsilon \neq 1$$

$$I = 1 - \left(\frac{1}{N}\sum_{i=1}^{N}(x_i/\mu)^{1-\epsilon}\right)^{1/(1-\epsilon)}$$

Welfare with inequality aversion $\in \geq 0$ Controls the degree of aversion towards inequality – Figure above when $\in$ is smaller the flatter is the curve .

Marginal Rate of Social Substitution

$$\frac{\partial W/\partial x_i}{\partial W/\partial x_j} = (x_j/x_i)^{\epsilon}.$$

If $\in = 0$, then marginal utility is fixed and I do not take inequality into account.

If $\in = 2$ and $x_i = 2xj$ then marginal social utility of giving x to i is 1/4 (of giving x to j) .If $\in = -\infty$, then utility is similar to Leontief type (Raws) , that is, what matters is the welfare of the poorest individual of society.

Deriving inequality from the Social Welfare Function

Considering Sen's welfare function $W = V(x_1,\ldots x_N) = \mu V(\frac{x_1}{\mu},\ldots,\frac{x_N}{\mu})\mu(1-I)$, where I used the hypothesis of first degree homogeneity (HG1) of function V, for the proportional change in all x's have the same proportional effect on the sum. Be the function of social welfare additive (Atkinson):

$$W = \frac{1}{N}\sum_{i=1}^{N}\frac{x_i^{1-\varepsilon}}{1-\varepsilon}, \varepsilon \neq 1$$

We shall verify if this function is HG1:

$$W(\lambda x) = \frac{1}{N}\sum_{i=1}^{N}\frac{(\lambda x_i)^{1-\varepsilon}}{1-\varepsilon} = \lambda^{1-\varepsilon}\frac{1}{N}\sum_{i=1}^{N}\frac{x_i^{1-\varepsilon}}{1-\varepsilon} = \lambda^{1-\varepsilon}W(x)$$

So, for W to be HG1, we have to raise to $1/(1-\varepsilon)$. So, we would have:

$$W^*(\lambda x) = [W(\lambda x)]^{\frac{1}{1-\varepsilon}} = [\lambda^{1-\varepsilon} W(x)]^{\frac{1}{1-\varepsilon}} = \lambda [W(x)]^{\frac{1}{1-\varepsilon}} = \lambda W^*(x)$$

using $I = 1 - \left[\frac{1}{N}\sum_{i=1}^{N}(x_i/\mu)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$ we will have a inequality measure associated with this W*(x). Verifying HG1 using Sen's formula:

$$W^*(\lambda x) = \mu\left[\frac{1}{N}\sum_{i=1}^{N}\frac{(\lambda x_i/\mu)^{1-\varepsilon}}{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} = \mu\left[\frac{1}{N}\sum_{i=1}^{N}\frac{\lambda^{1-\varepsilon}(x_i/\mu)^{1-\varepsilon}}{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} = \mu\left[\frac{\lambda^{1-\varepsilon}}{\mu^{1-\varepsilon}}\frac{1}{N}\sum_{i=1}^{N}\frac{x_i^{1-\varepsilon}}{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} = \lambda\left[\frac{1}{N}\sum_{i=1}^{N}\frac{x_i^{1-\varepsilon}}{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} = \lambda W^*(x)$$

So, the inequality measure is $I = 1 - \left[\frac{1}{N}\sum_{i=1}^{N}(x_i/\mu)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$ for $\varepsilon \neq 1$

For $\varepsilon = 1$, the resolution is direct. See:

$$\ln W = \frac{1}{N}\sum_{i=1}^{N}\ln x_i = \frac{1}{N}\ln\prod_{i=1}^{N}x_i = \ln\left(\prod_{i=1}^{N}x_i\right)^{\frac{1}{N}}, \varepsilon = 1$$

$$W = \left(\prod_{i=1}^{N}x_i\right)^{\frac{1}{N}}$$

Once this function is HG1, we can propose an inequality measure directly:

$$I = 1 - \left(\prod_{i=1}^{N}x_i\right)^{\frac{1}{N}}$$

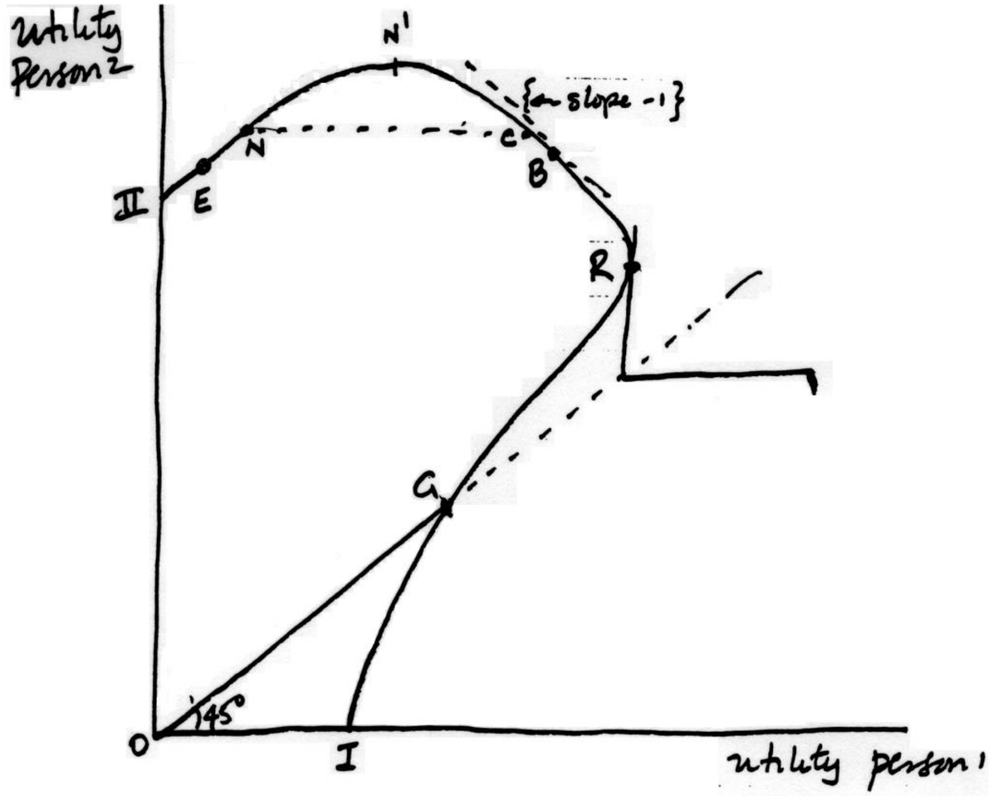
Verifying HG1 using Sen's formula:

$$W(\lambda x) = \mu\left(\prod_{i=1}^{N}\lambda x_i/\mu\right)^{\frac{1}{N}} = \lambda^{N/N}\mu\frac{1}{\mu^{N/N}}\left(\prod_{i=1}^{N}x_i\right)^{\frac{1}{N}} = \lambda W(x)$$

From: Social Welfare Function **To: Inequality (For $\varepsilon = 1$)**

$$W = \left(\prod_{i=1}^{N}x_i\right)^{\frac{1}{N}} \qquad I = 1 - \left(\prod_{i=1}^{N}x_i\right)^{\frac{1}{N}}$$

Social from Individual Welfare Functions: Examples of u(x) (from Deaton's class notes):



2 types of individuals: 1 e 2. Assume that all points between I and II are possible.

Betham: Max $W = \sum_i u^i$ where $u^i = u(p,I^i)$

Redistribute income $M = \sum_i I^i$ to Max in (eg) for to 2 people: $u(p,I^1)+u(p,M-I)$

$$(\partial u/\partial I^1)\partial I^1 + (\partial u/\partial I^2)\frac{\partial I^2}{\partial I^1} dI^1 = 0$$

Umg Income$_1$ Umg Income$_2$

$$\underbrace{-\frac{Mu^1}{Mu^2}}_{\text{Slope}} = \frac{\partial I^2}{\partial I^1} = -1 \Big\} \text{ Leaves you at point "B"}$$

Rawls: Max { Min (x_1, ... x_n)} min(u^h) takes you to point "R".
Vichery: If one person is uncertain of its position, then choose to maximize expected utility $\sum_h u^h / H$ (Again point B) Vichery is neutral to risk and Rawls has infinitely risk aversion.

Egalitarian: $W = A - \gamma \left| u^1 - u^2 \right|$ for A>0, γ>0 leave you above $\overline{OG}$ line. Is not-paretian

Paternalist: Also not paretian, individual utilities do not influence social welfare function.

OTHER INEQUALITY MEASURES

Gini Index – It can be derived directly from social welfare function with weight structure equal to (1- F(x)) of individual incomes, see above) reaching aggregated $\mu\ (1 - \delta)$ where μ is the average income, δ the Gini coefficient and ק ~ [0 , 1] from Sen (1976). Or more generally, $\mu\ (1 - \delta)^{\rho}$ where ρ is the inequality aversion parameter from Graff (1981). Gini is popular due to its tradition, scale and intuition. Disadvantages: Does not change much, low sensibility to bottom income changes and not very adapted to decompositions.

Bottom 40% Share in total income – Shared Prosperity as in Goal 10 of Sustainable Development Goals (SDGs). Sensitive by construction to the lower end of income distribution. Derived directly from Social Welfare function but does not follow the principle of transfers.

Theil T, Theil L and J-Divergence Indexes – Belong to the family of entropy measures (other than social welfare function deduction approach). Highly decomposable but its range is awkward it can be fixed through its dual. Later 2 measures does not allow null incomes

Variance of Logs $V_{\log} = \frac{1}{N}\sum(\overline{\log y} - \log y_i)$

Advantages: Insensitive to scale, Allows disaggregation
Disadvantages: Do not exist for yi=0, Little sensitive on the top, - Does not follow the transference principle (critic less relevant in practice for inequality, more for concentration measures). Decompositions works out nicely in a log-linear regression framework

Other measures (statistical approach, less used in economics).

Average Deviation $DM = \sum \frac{|y_i - \mu|}{N\mu}$ Disadvantage: do not follows Pigou-Dalton

Coefficient of Variation

$$CV = \frac{1}{\mu}\left[\frac{1}{N}\sum (y_i - \mu)^2\right]^{1/2}$$

Interquartilic Amplitude :

$\frac{\text{Income 75\%}}{\text{Income 25\%}}$ Do not follow the principle of transfers. Transfering from a low quartile for someone poorer can raise inequality.

Total amplitude

$\beta_1 = \frac{[\text{Max } y_i - \text{Min } y_i]}{\mu}$ or $\beta_2 = \frac{\text{Max } y_i}{\text{Min } y_i}$ disadvantage: very sensitive to *outliers*

***Palma Ratio**:* It is the ratio of the richest 10% of the population's share of income divided by the poorest 40%'s share. Recent but already popular.