SOCIAL WELFARE

References:
* Deaton (1997), Chapter 3, section 3.1 (section 3.2 will be used on the Poverty part)
* Based on Atkinson’s classic “On The Measurement of Inequality” (1970)

Social Welfare Function (SWF)

\[ W = V(x_1, x_2, \ldots, x_N) \]
SWF function is a sum across individuals (typically of per capita expenditures or income).

**Properties of BES Functions**

- **Pareto Optimum** – V is increasing (non-decreasing) in its arguments. If one gets better and nobody worse it increases – to accommodate poverty measures (truncated BES functions) we adopt non-decreasing function.
- **Symmetry or Anonymity** – BES depends on individual welfare levels and not their identity.
- **Principle of Transfers** (Pigou – Dalton) – For a given total X, BES function will be at its maximum point when inequality will be at the same time at its minimum, conditioned to the average (when OC’S are equal) – express an equity preference. Ignore any kind of restrictions on allocations and incentives effects.
  Decreasing marginal utility (quasi-concavity or more general S–concavity). If x₁ and x₂ are lists of x’s and if V (x₁) = V (x₂) then \( \lambda \cdot x₁ + (1 - \lambda) \cdot x₂ \) for a \( \lambda \in [0, 1] \) will have a higher value or equal to the original allocations.

**SOCIAL WELFARE AND INEQUALITY**

If V is homogeneous of the 1st degree

\[
W = \mu V \left( \frac{x₁}{\mu}, \ldots, \frac{xₙ}{\mu} \right)
\]

Separate inequality and mean effects

If we normalize units as V (1,1,...,1) = 1

When there is perfect equality, that is, everybody have individual level of welfare, social welfare has the same value.

\[
W = \mu (1 - I)
\]

By the transfers principle, inequality is the cost that makes the value of social welfare falls below the perfect equality point.

X*is the equivalent of x equally distributed

BC/OC is the geometric measure of inequality proposed by Atkinson.

One advantage of this approach is to differentiate inequality and social welfare. It is consistent with poverty.
Welfare with inequality aversion $\varepsilon \geq 0$ Controls the degree of aversion towards inequality – Figure above when $\varepsilon$ is smaller the flatter is the curve.

**Marginal Rate of Social Substitution**

$$\frac{\partial W}{\partial x_i} = (x_j/x_i)^{\varepsilon}$$

If $\varepsilon = 0$, then marginal utility is fixed and I do not take inequality into account.

If $\varepsilon = 2$ and $x_i = 2x_j$ then marginal social utility of giving $x$ to $i$ is $1/4$ (of giving $x$ to $j$). If $\varepsilon = -\infty$, then utility is similar to Leontief type (Raws), that is, what matters is the welfare of the poorest individual of society.

**Deriving inequality from the Social Welfare Function**

Considering Sen’s welfare function $W = V(x_1, ..., x_N) = \mu V(x_1/\mu, ..., x_N/\mu)\mu(1-I)$, where I used the hypothesis of first degree homogeneity (HG1) of function $V$, for the proportional change in all $x$’s have the same proportional effect on the sum. Be the function of social welfare additive (Atkinson):

$$W = \frac{1}{N} \sum_{i=1}^{N} x_i^{1-\varepsilon}, \varepsilon \neq 1$$

We shall verify if this function is HG1:

$$W(\lambda x) = \frac{1}{N} \sum_{i=1}^{N} (\lambda x_i)^{1-\varepsilon} = \lambda^{1-\varepsilon} \frac{1}{N} \sum_{i=1}^{N} x_i^{1-\varepsilon} = \lambda^{1-\varepsilon} W(x)$$

So, for $W$ to be HG1, we have to raise to $1/(1-\varepsilon)$. So, we would have:
\( W^*(\lambda x) = [W(\lambda x)]^{1/1-\varepsilon} = [\lambda^{1-\varepsilon} W(x)]^{1/1-\varepsilon} = \lambda W^*(x) \)

using \( I = 1 - \left[ \frac{1}{N} \sum_{i=1}^{N} (x_i / \mu)^{1-\varepsilon} \right]^{1/1-\varepsilon} \) we will have an inequality measure associated with this \( W^*(x) \). Verifying HG1 using Sen’s formula:

\[
W^*(\lambda x) = \mu \left[ \frac{1}{N} \sum_{i=1}^{N} \left( \frac{\lambda x_i}{\mu} \right)^{1-\varepsilon} \right]^{1/1-\varepsilon} = \mu \left[ \frac{1}{N} \sum_{i=1}^{N} \lambda^{1-\varepsilon} (x_i / \mu)^{1-\varepsilon} \right]^{1/1-\varepsilon} = \mu \left[ \frac{1}{N} \sum_{i=1}^{N} (x_i / \mu)^{1-\varepsilon} \right]^{1/1-\varepsilon} = \lambda \left[ \frac{1}{N} \sum_{i=1}^{N} (x_i / \mu)^{1-\varepsilon} \right]^{1/1-\varepsilon} = \lambda W^*(x)
\]

So, the inequality measure is \( I = 1 - \left[ \frac{1}{N} \sum_{i=1}^{N} (x_i / \mu)^{1-\varepsilon} \right]^{1/1-\varepsilon} \) for \( \varepsilon \neq 1 \)

For \( \varepsilon = 1 \), the resolution is direct. See:

\[
\ln W = \frac{1}{N} \sum_{i=1}^{N} \ln x_i = \frac{1}{N} \ln \prod_{i=1}^{N} x_i = \ln \left( \prod_{i=1}^{N} x_i \right)^{1/N}, \varepsilon = 1
\]

\[
W = \left( \prod_{i=1}^{N} x_i \right)^{1/N}
\]

Once this function is HG1, we can propose an inequality measure directly:

\[
I = 1 - \left( \prod_{i=1}^{N} x_i \right)^{1/N}
\]

Verifying HG1 using Sen’s formula:

\[
W(\lambda x) = \mu \left( \prod_{i=1}^{N} \lambda x_i / \mu \right)^{1/N} = \lambda^{N/1-N} \mu^{1/N} \left( \prod_{i=1}^{N} x_i \right)^{1/N} = \lambda W(x)
\]

**From: Social Welfare Function**

\[
W = \left( \prod_{i=1}^{N} x_i \right)^{1/N}
\]

**To: Inequality (For \( \varepsilon = 1 \))**

\[
I = 1 - \left( \prod_{i=1}^{N} x_i \right)^{1/N}
\]
Social Welfare Function Examples

2 types of individuals: 1 e 2. Assume that all points between I and II are possible.

**Betham:** Max \( W = \sum_i u^i \) where \( u^i = u(p, I^i) \)
Redistribute income \( M = \sum_i I^i \) to Max in (eg) for to 2 people: \( u(p, I^1) + u(p, M - I) \)
\[
\left( \frac{\partial u}{\partial I^1} \right) \delta I^1 + \left( \frac{\partial u}{\partial I^2} \right) \frac{\partial I^2}{\partial I^1} dI^1 = 0
\]

\( Umg \) Income\(_1\) \quad \text{Umg Income\(_2\)}

\[
- \frac{\mu u^1}{\mu u^2} = \frac{\partial I^2}{\partial I^1} = -1 \quad \text{Leaves you at point “B”}
\]

**Rawls:** Max \{ Min \((x_1, \ldots x_n)\) \}
\( \min(u^h) \) takes you to point “R”.

**Vichery:** If one person is uncertain of its position, then choose to maximize the expected utility \( \sum_h u^h/H \) (Again takes you to point B)
Vichery is neutral to risk and Rawls has infinitely risk aversion.
**Egalitarian:** $W = A - \gamma |u^1 - u^2|$ for $A > 0, \gamma > 0$ leave you above $OG$ line.

Is not-paretian

**Paternalist:** Also not paretian, individual utilities do not influence social welfare function.

Sen: BES function in Sen (1976) is $\mu (1 - \delta)$

where $\mu$ is the average income, $\delta$ the Gini coefficient of the poor and $\gamma \sim [0, 1]$.

**OTHER INEQUALITY MEASURES**

- Gini Index – It can be derivated directly from social welfare function with weight structure equal to $1 - F(x)$ of individual incomes
- Theil T, Theil L and J-Divergence Indexes – Belong to the family of entropy measures

**Logs Variance**

$V \log = \frac{1}{N} \sum \log y - \log y_i$

Advantages: Insensitive to scale, Allows disaggregation

Disadvantages: Do not exist for $yi=0$, Little sensitive on the top

- Do not follow the transference principle (critic less relevant in practice for inequality, more for concentration measures)

**Other measures** (statistical approach, less used in economics).

**Average Deviation** $DM = \frac{\sum |y_i - \mu|}{N\mu}$ Disadvantage: do not follows Pigou-Dalton

**Coefficient of Variation**

$CV = \frac{1}{\mu} [\frac{1}{N} \sum (y_i - \mu)^2]^{1/2}$

**Interquartilic Amplitude**:

\[
\frac{\text{Income 75%}}{\text{Income 25%}}
\]

Do not follow the principle of transfers. Transfering from a low quartile for someone poorer can raise inequality.

**Other amplitudes**

$\beta_1 = \frac{[\text{Max } y_i - \text{Min } y_i]}{\mu}$ or $\beta_2 = \frac{\text{Max } y_i}{\text{Min } y_i}$

disadvantage: very sensitive to outliers