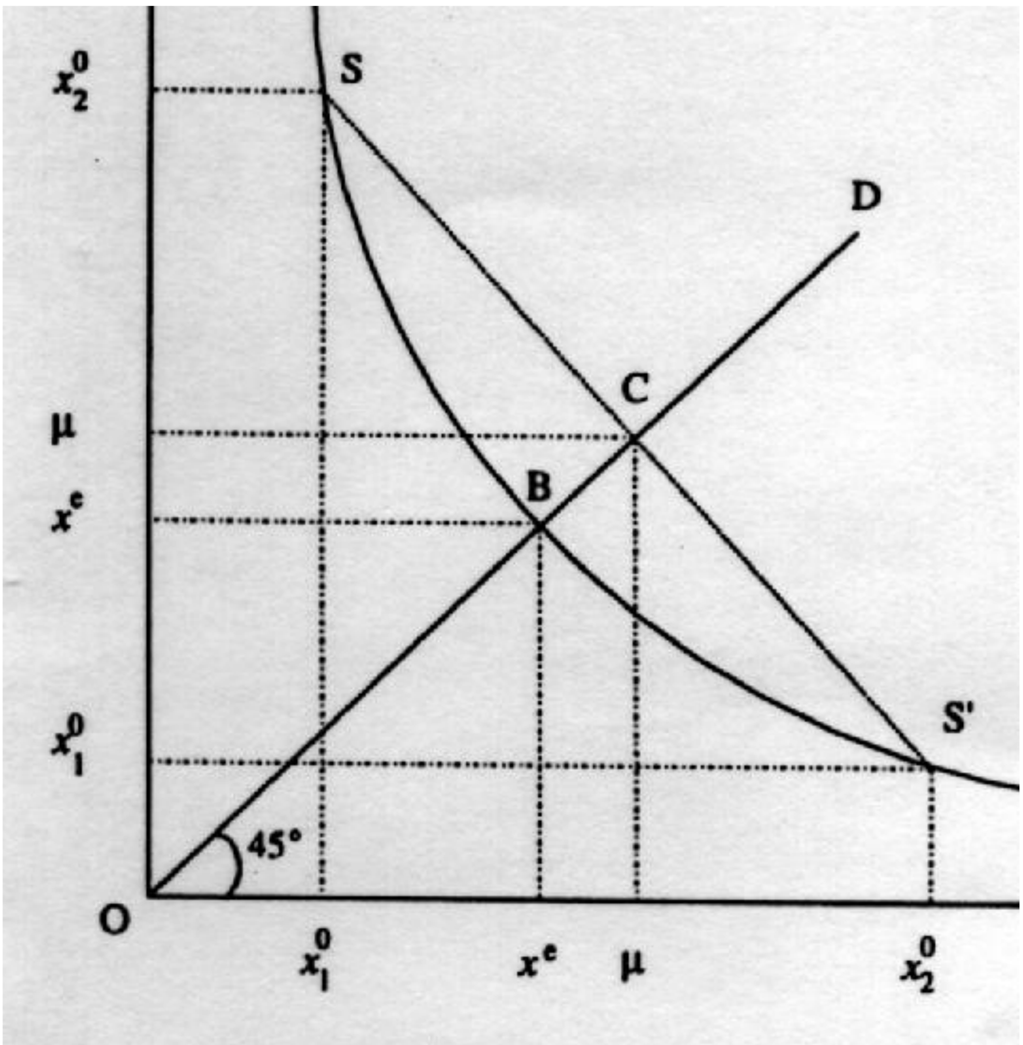
*SOCIAL WELFARE

References:
*Deaton (1997) , Chapter 3, section 3.1 (section 3.2 will be used on the Poverty part)
₴ Based on Atkinson's classic "On The Measurement of Inequality" (1970)

Social Welfare Function (SWF)

$$W = V(x_1, x_2, \ldots, x_N)$$



SWF function is a sum across individuals (typically of per capita expenditures or income) .

Properties of BES Functions

- Pareto Optimum – V is increasing (non decreasing) in its arguments. If one gets better and nobody worse it increases – to accommodate poverty measures (truncated BES functions) we adopt non decreasing function.
- Symmetry or Anonymity – BES depends on individual welfare levels and not their identity.
- Principle of Transfers (Pigou – Dalton) – For a given total X, BES function will be at it maximum point when inequality will be at the same time at its minimum, conditioned to the average (when OC' S are equal) – express an equity preference. Ignore any kind of restrictions on allocations and incentives effects.
  Decreasing marginal utility (quasi-concavity or more general S – concavity) . If $\mathbf{x_1}$ and $\mathbf{x_2}$ are lists of x's and if V ($\mathbf{x_1}$) =V ($\mathbf{x_2}$) then $\lambda\ \mathbf{x_1} + (1 - \lambda)\ \mathbf{x_2}$ for a λ – [0 , 1] will have a higher value or equal to the original allocations.

SOCIAL WELFARE AND INEQUALITY

If V is homogeneous of the 1st degree

$$W = \mu V\left(\frac{x_1}{\mu}, \ldots, \frac{x_N}{\mu}\right)$$

Separate inequality and mean effects

If we normalize units as V (1,1,...,1) = 1
When there is perfect equality, that is, everybody have individual level of welfare, social welfare has the same value.

$$W = \mu (1 - I)$$

By the transfers principle, inequality is the cost that makes the value of social welfare falls below the perfect equality point.

$X^e$is the equivalent of x equally distributed
BC/OC is the geometric measure of inequality proposed by Atkinson.
One advantage of this approach is to differentiate inequality and social welfare. It is consistent with poverty.

From: Social Welfare Function **To: Inequality**

$$W = \frac{1}{N}\sum_{i=1}^{N}\frac{x_i^{1-\epsilon}}{1-\epsilon}, \quad \epsilon \neq 1$$

$$I = 1 - \left(\frac{1}{N}\sum_{i=1}^{N}(x_i/\mu)^{1-\epsilon}\right)^{1/(1-\epsilon)}$$

Welfare with inequality aversion $\in \geq 0$ Controls the degree of aversion towards inequality – Figure above when $\in$ is smaller the flatter is the curve .

Marginal Rate of Social Substitution

$$\frac{\partial W/\partial x_i}{\partial W/\partial x_j} = (x_j/x_i)^{\in} \qquad = (x_j/x_i)^{\in}$$

If $\in = 0$, then marginal utility is fixed and I do not take inequality into account.

If $\in = 2$ and $x_i = 2x_j$ then marginal social utility of giving x to i is 1/4 (of giving x to j) .If $\in = -\infty$, then utility is similar to Leontief type (Raws) , that is, what matters is the welfare of the poorest individual of society.

Deriving inequality from the Social Welfare Function

Considering Sen's welfare function $W = V(x_1,\dots x_N) = \mu V(\frac{x_1}{\mu},\dots,\frac{x_N}{\mu})\mu(1-I)$, where I used the hypothesis of first degree homogeneity (HG1) of function V, for the proportional change in all x's have the same proportional effect on the sum. Be the function of social welfare additive (Atkinson):

$$W = \frac{1}{N}\sum_{i=1}^{N}\frac{x_i^{1-\varepsilon}}{1-\varepsilon}, \varepsilon \neq 1$$

We shall verify if this function is HG1:

$$W(\lambda x) = \frac{1}{N}\sum_{i=1}^{N}\frac{(\lambda x_i)^{1-\varepsilon}}{1-\varepsilon} = \lambda^{1-\varepsilon}\frac{1}{N}\sum_{i=1}^{N}\frac{x_i^{1-\varepsilon}}{1-\varepsilon} = \lambda^{1-\varepsilon}W(x)$$

So, for W to be HG1, we have to raise to $1/(1-\varepsilon)$. So, we would have:

$W^*(\lambda x) = [W(\lambda x)]^{\frac{1}{1-\varepsilon}} = [\lambda^{1-\varepsilon} W(x)]^{\frac{1}{1-\varepsilon}} = \lambda [W(x)]^{\frac{1}{1-\varepsilon}} = \lambda W^*(x)$

using $I = 1 - \left[\frac{1}{N} \sum_{i=1}^{N} (x_i / \mu)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$ we will have a inequality measure associated with this W*(x). Verifying HG1 using Sen's formula:

$$W^*(\lambda x) = \mu \left[\frac{1}{N} \sum_{i=1}^{N} \frac{(\lambda x_i / \mu)^{1-\varepsilon}}{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} = \mu \left[\frac{1}{N} \sum_{i=1}^{N} \frac{\lambda^{1-\varepsilon} (x_i / \mu)^{1-\varepsilon}}{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} = \mu \left[\frac{\lambda^{1-\varepsilon}}{\mu^{1-\varepsilon}} \frac{1}{N} \sum_{i=1}^{N} \frac{x_i^{1-\varepsilon}}{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} = \lambda \left[\frac{1}{N} \sum_{i=1}^{N} \frac{x_i^{1-\varepsilon}}{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} = \lambda W^*(x)$$

So, the inequality measure is $I = 1 - \left[\frac{1}{N} \sum_{i=1}^{N} (x_i / \mu)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$ for $\varepsilon \neq 1$

For $\varepsilon = 1$, the resolution is direct. See:

$$\ln W = \frac{1}{N} \sum_{i=1}^{N} \ln x_i = \frac{1}{N} \ln \prod_{i=1}^{N} x_i = \ln \left(\prod_{i=1}^{N} x_i \right)^{\frac{1}{N}}, \varepsilon = 1$$

$$W = \left(\prod_{i=1}^{N} x_i \right)^{\frac{1}{N}}$$

Once this function is HG1, we can propose an inequality measure directly:

$$I = 1 - \left(\prod_{i=1}^{N} x_i \right)^{\frac{1}{N}}$$

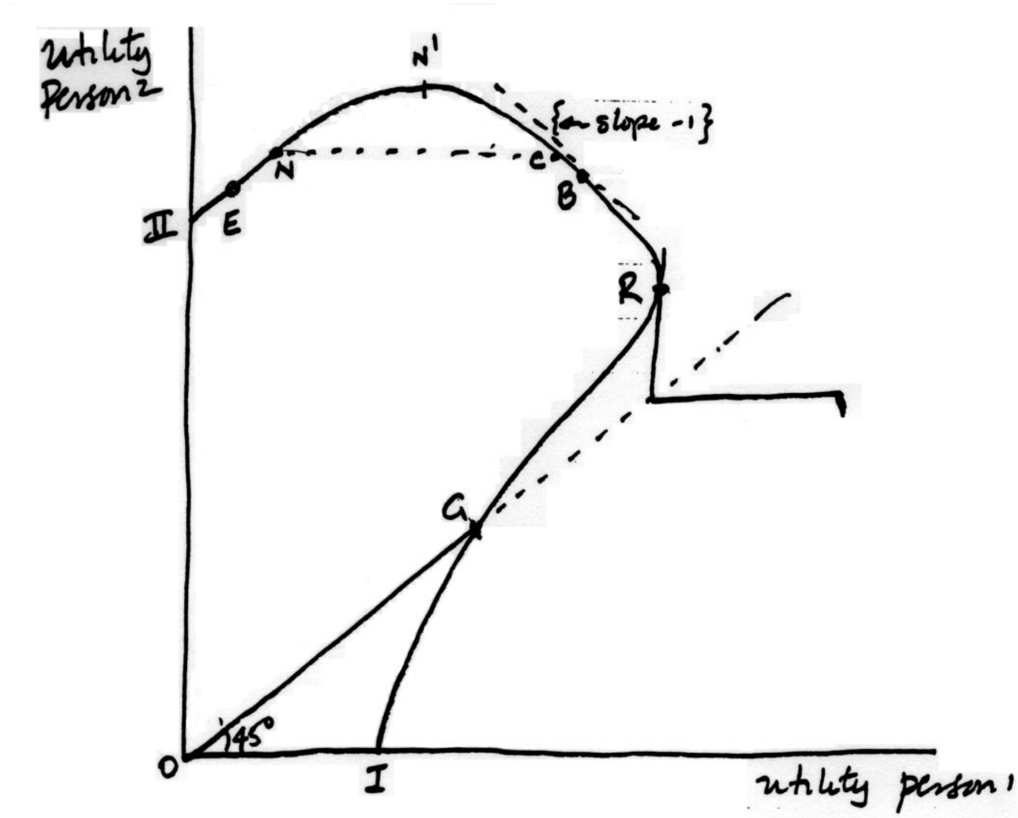
Verifying HG1 using Sen's formula:

$$W(\lambda x) = \mu \left(\prod_{i=1}^{N} \lambda x_i / \mu \right)^{\frac{1}{N}} = \lambda^{N/N} \mu \frac{1}{\mu^{N/N}} \left(\prod_{i=1}^{N} x_i \right)^{\frac{1}{N}} = \lambda W(x)$$

From: Social Welfare Function **To: Inequality (For $\varepsilon = 1$)**

$$W = \left(\prod_{i=1}^{N} x_i \right)^{\frac{1}{N}} \qquad I = 1 - \left(\prod_{i=1}^{N} x_i \right)^{\frac{1}{N}}$$

Social Welfare Function Examples



2 types of individuals: 1 e 2. Assume that all points between I and II are possible.

Betham: Max $W = \sum_i u^i$ where $u^i = u(p, I^i)$

Redistribute income $M = \sum_i I^i$ to Max in (eg) for to 2 people: $u(p,I^1)+u(p,M-I)$

$$(\partial u/\partial I^1)\partial I^1 + (\partial u/\partial I^2)\frac{\partial I^2}{\partial I^1} dI^1 = 0$$

Umg Income$_1$ Umg Income$_2$

$$\underbrace{-\frac{Mu^1}{Mu^2}}_{\text{Slope}} = \frac{\partial I^2}{\partial I^1} = -1 \Big\} \text{ Leaves you at point "B"}$$

Rawls: Max { Min (x_1, ... x_n)}
min(u^h) takes you to point “R”.

Vichery: If one person is uncertain of its position, then choose to maximize the expected utility $\sum_h u^h / H$ (Again takes you to point B)
Vichery is neutral to risk and Rawls has infinitely risk aversion.

Egalitarian: $W = A - \gamma\left|u^1 - u^2\right|$ for A>0, γ>0 leave you above $\overline{OG}$ line.
Is not-paretian

Paternalist: Also not paretian, individual utilities do not influence social welfare function.

Sen : BES function in Sen (1976) is μ (1 – δ)
where μ is the average income, δ the Gini coefficient of the poor and ק ~ [0 , 1].

OTHER INEQUALITY MEASURES

- Gini Index – It can be derivated directly from social welfare function with weight structure equal to 1- F(x) of individual incomes)
- Theil T, Theil L and J-Divergence Indexes – Belong to the family of entropy measures

Logs Variance

$$V\log = \frac{1}{N}\sum(\overline{\log y} - \log y_i)$$

Advantages: Insensitive to scale, Allows disaggregation
Disadvantages: Do not exist for yi=0, Little sensitive on the top, - Do not follow the transference principle (critic less relevant in practice for inequality, more for concentration measures)

Other measures (statistical approach, less used in economics).

Average Deviation $DM = \sum\frac{|y_i - \mu|}{N\mu}$ Disadvantage: do not follows Pigou-Dalton

Coefficient of Variation

$$CV = \frac{1}{\mu}\left[\frac{1}{N}\sum(y_i - \mu)^2\right]^{1/2}$$

Interquartilic Amplitude :

$$\frac{\text{Income } 75\%}{\text{Income } 25\%}$$

Do not follow the principle of transfers. Transfering from a low quartile for someone poorer can raise inequality. .

Other amplitudes

$$\beta_1 = \frac{[\mathrm{Max}\, y_i - \mathrm{Min}\, y_i]}{\mu} \text{ or } \beta_2 = \frac{\mathrm{Max}\, y_i}{\mathrm{Min}\, y_i}$$

disadvantage: very sensitive to *outliers*