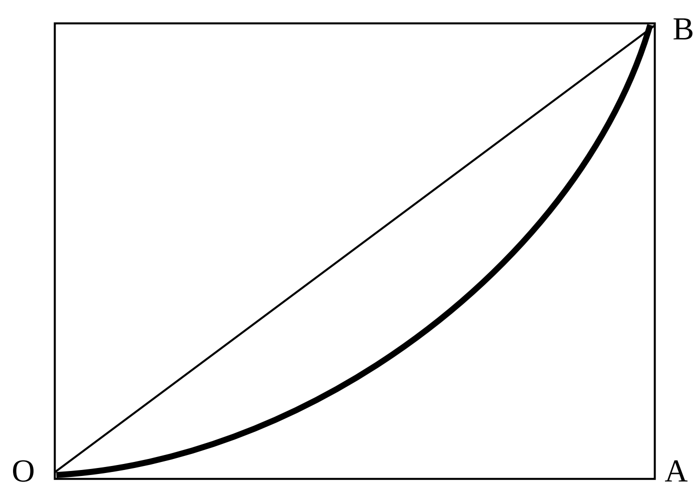
LORENZ CURVE

Lorenz curve is a simple graphic instrumental that allows the description of the income distribution in a given society, besides helping the ordering of different distributions departing from a welfare point of view (Max Otto Lorenz 1905).

Lorenz curve express the relation between the cumulative proportion of people with income at least equal to some specific value and the cumulative proportion of income received by these people. Lorenz curve is represented by a function L(P), which corresponds to the a fraction received by the p-th lower fraction of the population, when it is ordered by increasing income. The curve slope is always positive and convex, so L(0) = 0 and L(1) = 1.

The line L(p)=p is the line of perfect equity, corresponding to the OB line in the graph below. It is a situation which everybody receive the same amount of income. The line of extreme inequity corresponds to the lines AO and AB. It is a situation which everybody receive zero income except the richest person, who accumulate the total income.

Lorenz curve is always between the line of perfect equity and the line of extreme inequity. When nearest of the line of perfect equity, more egalitarian is the income distribution.

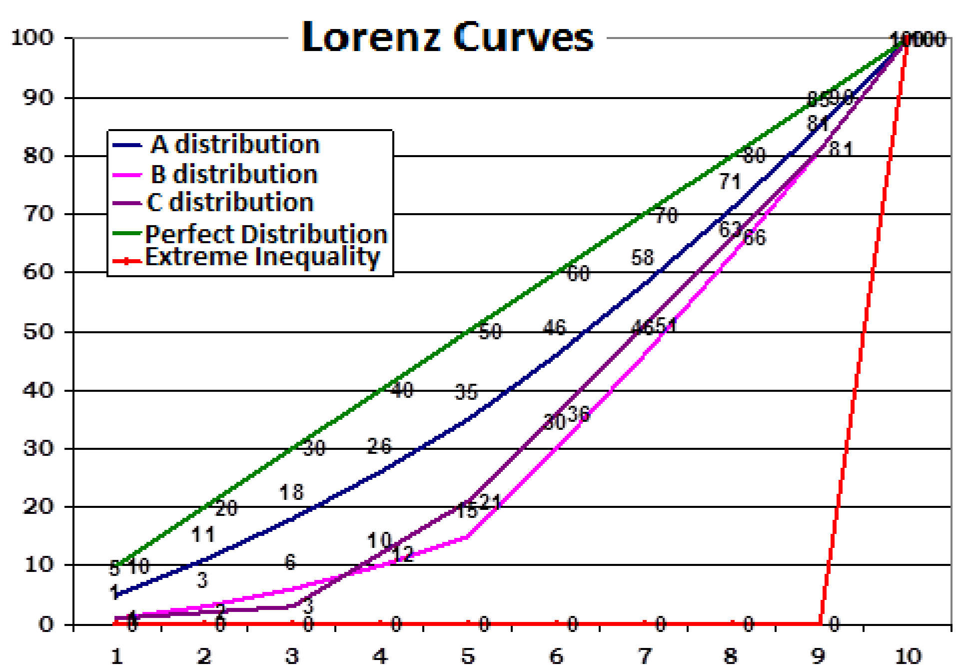


Lorenz Dominance

We say that the Lorenz curve of a distribution A dominates distribution of B if curve A is above curve B in all points of the distribution. In this case the one can say A is more equal than B. And if both distributions have the same mean, A is preferable to B.

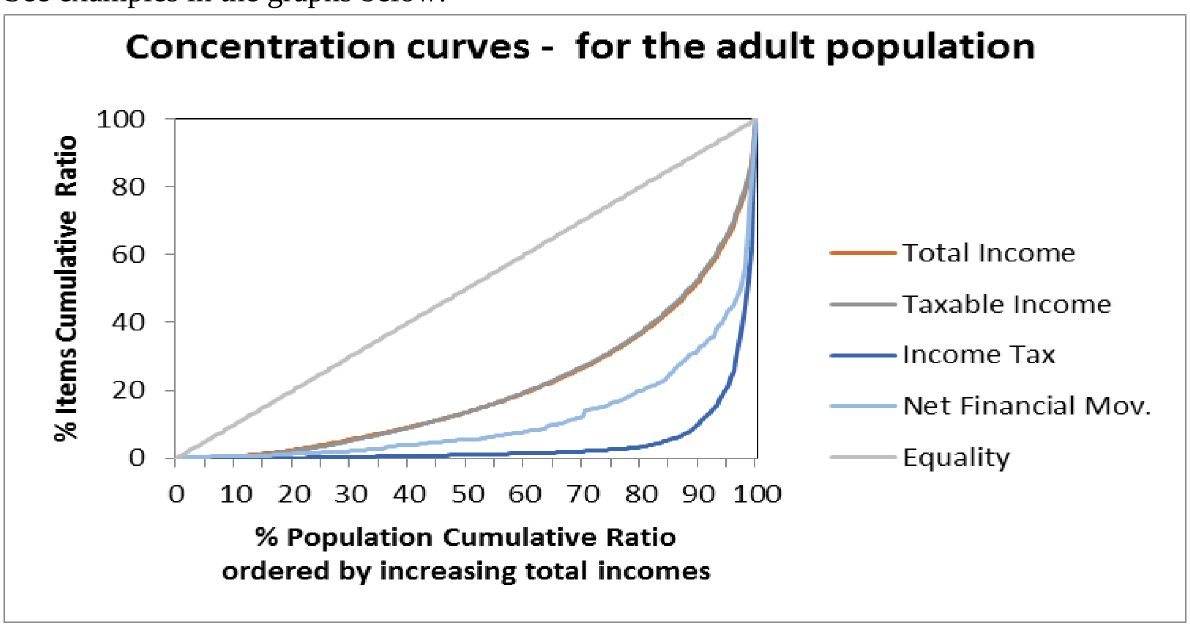
If there is an intersection point between two Lorenz curves, we can only make statements about stretches of the distribution. In this case, we can always find welfare functions that rank the distribution differently. In the example below, distribution of A dominate distribution of B and C, but distributions of B and C do not have any type of dominance when compared.

	A Distribution		B Distribution		C Distribution		Perfect Distribution		Extreme Inequity	
		Accumulated		Accumulated		Accumulated		Accumulated		Accumulated
1	**5**	5	**1**	1	**1**	1	**10**	10	**0**	0
2	**6**	11	**2**	3	**1**	2	**10**	20	**0**	0
3	**7**	18	**3**	6	**1**	3	**10**	30	**0**	0
4	**8**	26	**4**	10	**9**	12	**10**	40	**0**	0
5	**9**	35	**5**	15	**9**	21	**10**	50	**0**	0
6	**11**	46	**15**	30	**15**	36	**10**	60	**0**	0
7	**12**	58	**16**	46	**15**	51	**10**	70	**0**	0
8	**13**	71	**17**	63	**15**	66	**10**	80	**0**	0
9	**14**	85	**18**	81	**15**	81	**10**	90	**0**	0
10	**15**	100	**19**	100	**19**	100	**10**	100	**100**	100



Generalized Lorenz Curve - Lorenz curves in imply welfare dominance only when one compares distributions with the same mean, a rather restrictive hypothesis. Shorrocks (1983) and Kakwani (1984) developed a criterion to compare distributions with different mean. The Generalized Lorenz Curve is a modification of the Lorenz Curve in which the accumulated fraction of incomes up to each fraction of the population is multiplied by the average income of the distribution. Because of this multiplication, the generalized curve brings information about the form and level of the distribution, or the joint first two moments of distribution, such as the income distribution curve and its congeners of basic statistics. Lorenz Generalized Curve is represented by a function $L(\mu,P) = \mu L(P)$.
If the Lorenz Generalized Curve of the distribution of A is above the curve of the distribution of B in all points, then the welfare associated with A will be unequivocally superior then the welfare associated with B, for any symmetric welfare function (satisfy anonymity property) and quasi-concave (i.e. it satisfies Pigou Dalton property).

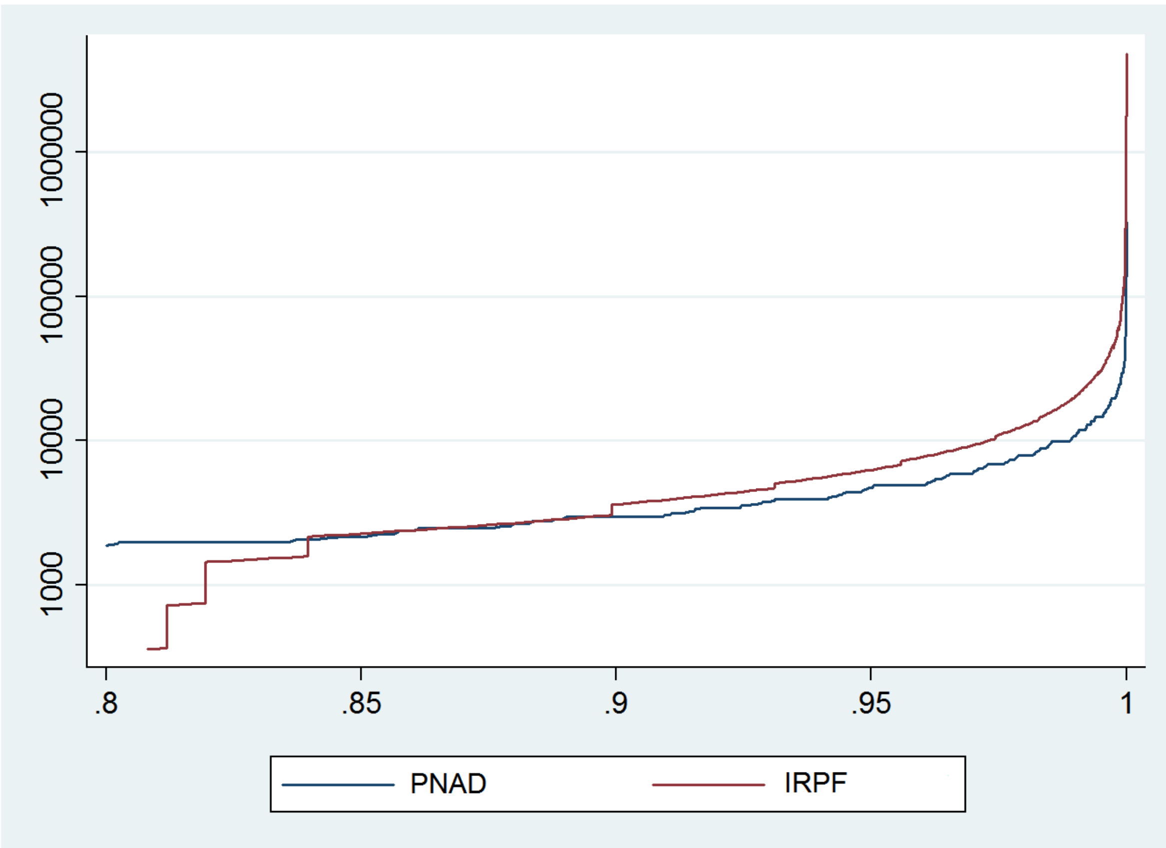
Concentration Curve - The Concentration Curves are a representation that bears similarities to the Lorenz Curve. However, while the latter refers to the distribution of a single variable throughout the population, the former are constructed from the distribution of two variables in the population In fact, the Lorenz Curve can be understood as a particular case of the Concentration Curve where the variable used in the ordering of the population and the output variable coincides. Similarly, the correspondence between Gini Index and the Lorenz Curve also appears in the relationship between the Concentration Curve and the Concentration Index. The difference is that the Gini varies between 0 and 1 while the Concentration Index varies between -1 and 1. If a certain attribute is more directed to the poor, for example conditional cash transfers, then the indicator is negative. See examples in the graphs below.



Source: POF 2008/09 IBGE microdata

Pen's Parade (Pen 1971) is a metaphor used to describe an income distribution. In it, income inequality is associated with inequality in people's height. This feature draws attention to the fact that if the height of people were proportional to their incomes, we would live in a society formed by a large mass of dwarves and a small elite of giants. There are other graphical representations of this distribution, usually a little different from the image of a crowd of dwarves. Imagine a population ordered by income, with the poorest people first and the richest afterwards. If we divide this population into one hundred equal parts, we will have one hundred percentiles, or twenty vintiles, and so on, that we refer here generally to the **income distribution curve**. That is nothing more than the representation of a cumulative distribution function (CDF) of incomes, with the axes reversed. Following the basic analogy with basic statistics there is still the density curve of income distribution.

Truncated Top Part of Adults Income Distribution PNAD X PIT income tax -log scale



Source: Hecksher et all (2017) – IRPF means PIT (Personal Income Tax) – both for 2014.

References:

₴ Lorenz, Max O. Methods of measuring the concentration of wealth. Publications of the American Statistical Association, v. 9, n. 70, p. 209-219, 1905.

₴ Pen, Jan. Income distribution: facts, theories, policies. New York: Praeger, 1971