Problem Set 1B  Social Welfare (inequality)

OBS: Problem set complement to Problem Set 1.

**Exercise 1**
Define what is a dual distribution of a random variable X with mean μ and distribution such that the value of some measure of inequality is M.

**Exercise 2**
Draw a Lorenz curve for a distribution suggested by the definition of a dual distribution.

**Exercise 3**
Show that the dispersion measure coefficient of variation attends the Pigou-Dalton condition.

Exercise 4
According to the general formula of an inequality measure

\[
S = \frac{1}{\varepsilon(1 - \varepsilon)} \left[ 1 - \frac{1}{n} \sum_{i=1}^{n} \left( \frac{x_i}{\mu} \right)^{1-\varepsilon} \right]
\]

Tell which measures of inequality it represents when: \(\varepsilon=-1, \varepsilon=0\) e \(\varepsilon=1\).

Exercise 5
Conceptually discuss the objectives, advantages and limitations of the following empirical techniques:

Income (mincerian log linear) equation (Coefficients, R2) – to decompose net contribution of different variables.

Difference in Difference Analysis – compare the bivariate with the multivariate approach

Exercise 7
Define and compare the uses of the following concepts using capsular formulas and graphs:

a) Lorenz curve and the Generalized Lorenz curve
b) Lorenz curve and Concentration curve
c) Gini index and Concentration ratio
d) Absolute and relative concepts of inequality

Exercise 6
Econometric Interpretation

i) Using the regression below, discuss the level and the evolution of the differentials in income by education in Brazil between 2001 and 2009. How to interpret the two coefficients in bold?

**Mincerian Equation (Log-Linear) of Individual Income with schooling and year as interactive variables**
Exercise 8

Using the Atkinson (1970) approach, derive an inequality measure for the share of the 10% richest in income (obs: use the shared prosperity framework)

Exercise 9

Comment, using capsular formulas and graphs:

a) The most popular inequality measures, like the Gini and the Theil-T indexes, are not very useful to discuss redistributive anti-poverty measures. Explain

b) The advantage of the J-divergence over the Theil-T is to allow decompositions between and within groups and also to incorporate null incomes. Explain

c) The advantage of the J-divergence over the Gini and the Theil-T indexes is to allow to decompose inequality in non negative income shares contributions. Explain

d) To introduce new 0s in the distribution as a way to proceed with maximum inequality so is adding top incomes. Use the dual formula to show how to introduce both in income inequality.

e) If female earnings are initially a fraction F% of males and if they grow f% faster per year how long at this pace it would take to eliminate the gender gap?

f) Show using the social welfare function proposed by Atkinson that mean income is a special case.

g) What is your favorite measure of income inequality? Justify

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### Estimated Regression Coefficients

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We omitted other variables