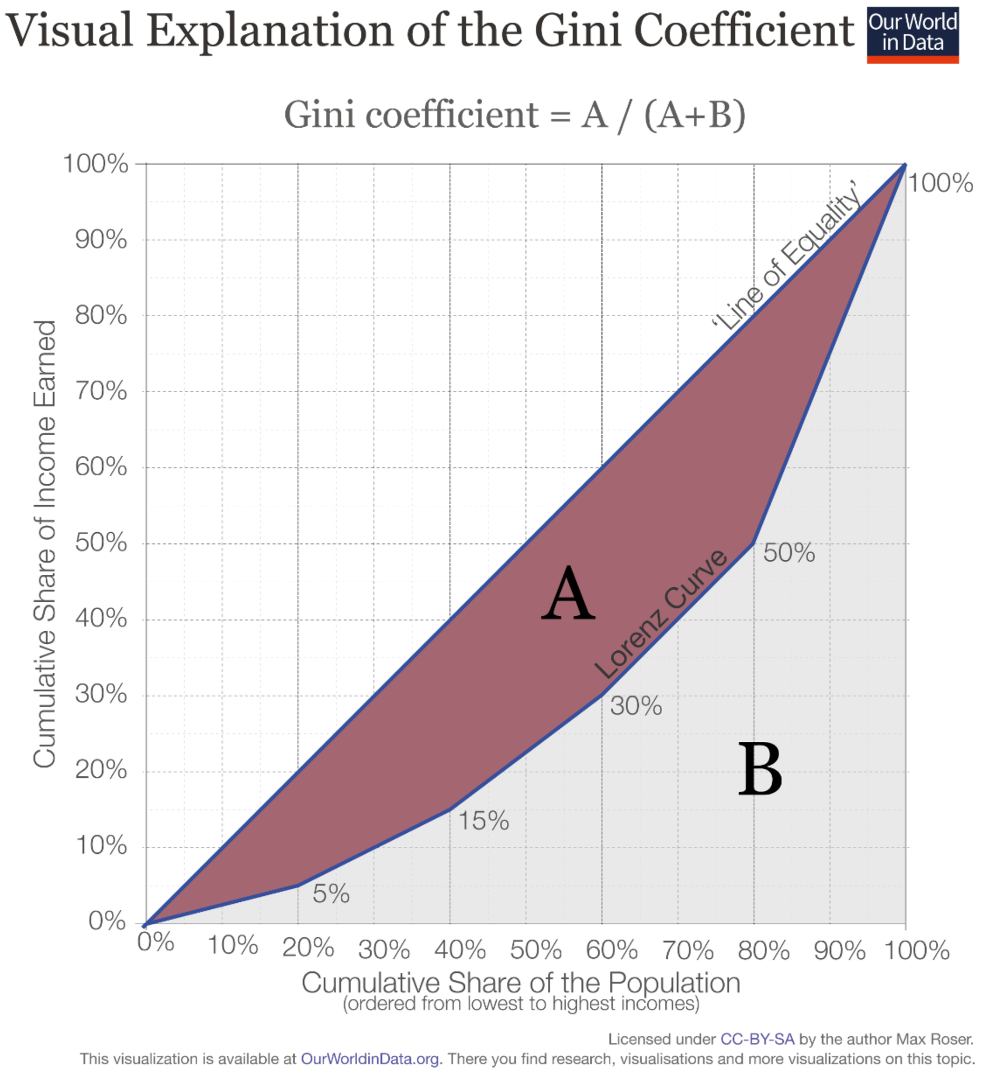
Gini Index

Geometric Interpretation – The Gini index corresponds to one minus two times the area between the Lorenz curve and the line of perfect equality area, or the ratio of areas A/A+B indicated in the graph below.



Gini Index - The direct calculation of the Gini Index from the Lorenz Curve demonstrated below in another advantage and explanation for its popularity. Gini (δ) ranges from 0 to 1.[1]

Example of Aggregated Level Welfare Function (BES) based on the Gini

BES function proposed by Sen (1976): $\mu\,(1 - \delta)$. More generally, the function proposed by Graff (1981): $\mu\,(1 - \delta)^{\rho}$, where $\rho \in [0, 1]$ (ρ is an inequality aversion parameter)

[1] OBS: The ratio between the Gini index of a given good and the Gini index of total income (i.e., the Gini) is equal to elasticity of income of that respective expenditure.

Analytical Interpretation

The Gini is an inequality index, corresponding to the ratio between the average of the absolute deviations of the incomes of all the people in the sample and twice the average. Once there are $\frac{N\,(N-1)}{2}$ distinct pairs of people in the sample, Gini's formula is:

$$\gamma = \frac{1}{\mu N(N-1)} \sum_{i>j}^{N} \sum_{j}^{N} |x_i - x_j|$$

- Perfect Equality: when all individuals have the same income, $x_i = \mu \forall i$, the sum above is equal to zero and Gini is also equal to zero.

- Perfect Inequality: when one individual has all the wealth ($N\mu$), we have $N - 1$ pairs with absolute deviations equal to $N\mu$, while the rest of the pairs have null deviations. Therefore, Gini is equal to one.

Alternative Formula

$$\gamma = \frac{N+1}{N-1} - \frac{2}{N\,(N-1)\mu} \sum_{i=1}^{N} \rho_i \, x_i$$

Where ρ_i is the ranking in the decreasingly ordered distribution. Note that the poorest individuals receive highest weights. Once it has only one sum, its computational cost is lower than of the previous formula

Advantages

- Simplicity
- Insensitive to scale

Disadvantages

- It is not fully decomposable

Reference:

₴Gini, Corrado. Variabilità e mutabilità: contributo allo studio delle distribuzioni e delle relazioni statistiche. (Fasc. I.). Tipogr. di P. Cuppini, 1912.