Appendix 1: Alternative Methodology to Decompose Productivity

Schooling is a major factor that has an impact on productivity. It is generally true that the higher the level of schooling an individual possesses, the greater is his/her productivity (or labour earnings per hour). The relationship between productivity and schooling is not exact. There can be many unexplained factors that have an impact on productivity. A household consists of working and non-working members. Schooling of non-working members may not be relevant to explaining labour productivity in the household. Hence we account for per capita years of schooling of only working members within household. Suppose $s^*$ is the per capita years of schooling of the working members in household. Using this variable, we fit the following regression model that explains productivity:

$$\log(\xi) = \alpha + \beta_1 \log(s^*) + \beta_2 \log(s^*)^2 + \log(u)$$  \hspace{1cm} (A1)

where $u$ is the error term which represents the aggregate impact of omitted variables from the model. Note that this regression equation can be estimated at household level using the weighted least squares method with weights being equal to population households represented by each sample household in the survey. Suppose $\hat{\alpha}, \hat{\beta}_1$ and $\hat{\beta}_2$ are the estimates of the model, which on substituting in (A1) gives

$$\log(\hat{\xi}) = \log(\hat{\xi}_s) + \log(\hat{u})$$  \hspace{1cm} (A2)

where

$$\log(\hat{\xi}_s) = \hat{\alpha} + \hat{\beta}_1 \log(s^*) + \hat{\beta}_2 \log(s^*)^2$$ \hspace{1cm} and \hspace{1cm} $\log(\hat{u}) = \log(\hat{\xi}) - \log(\hat{\xi}_s)$.

Using (A2), we can write the growth rates and the pro-poor growth rates in
productivity in an additive fashion as

\[ \gamma(\hat{z}) = \gamma(\hat{z}_s) + \gamma(\hat{u}) \]  

(A3)

and

\[ \gamma^*(\hat{z}) = \gamma^*(\hat{z}_s) + \gamma^*(\hat{u}) \]  

(A4)

which show that growth rates in productivity can be decomposed as the sum of two components: the first component is the impact of schooling and the second component is the aggregate effect of all the unexplained factors.

Subtracting (A3) from (A4) gives

\[ g^*(\hat{z}) = g^*(\hat{z}_s) + g^*(\hat{u}) \]  

(A5)

This equation provides the contributions of schooling and other unexplained variables to the growth rate of inequality in productivity. If, for instance, \( g^*(\hat{z}_s) \) is positive (or negative), this means that changes in schooling contribute to a decrease (or increase) in inequality in per capita income. Schooling can impact inequality in productivity through two factors. The first factor is the change in inequality of years of schooling and the second factor relates to returns from education. The first component in (A6) is the total effects of both factors.

There could be various factors that have impacts on productivity. These factors might include years of schooling, returns to schooling, gender, experience, and so forth. In this study, we particularly look into years of schooling and returns to schooling. According to our regression analysis, the years of schooling are able to explain per capita productivity by almost 93-95 percent: R-square of the regression model varies between 0.93 and 0.95. This suggests that the years of schooling could be a prime factor that explains per capita productivity.

Table A1 examines growth rates of years of schooling over the period with which we are concerned. Note that the number of years of schooling differ from one household to another as they are adjusted for household size. In the table per capita years of
schooling are presented for both all members and only working members within household. From the results we find an overall increase in years of schooling but a higher increase for the poor. As a result, more years of schooling have contributed to a fall in inequality of years of schooling over the period, which is sharper in the second period, 2001-04. The pro-poorness of schooling is far greater in the second period compared to the first period. In addition, the results highlight that the degree of pro-poorness of schooling of working members is higher than that of all members within household.

Table A1: Growth rates of per capita years of schooling

<table>
<thead>
<tr>
<th>Period</th>
<th>All members</th>
<th>Working members</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual growth rate</td>
<td>Pro-poor growth rate</td>
</tr>
<tr>
<td>1995-96</td>
<td>5.28</td>
<td>7.97</td>
</tr>
<tr>
<td>1996-97</td>
<td>1.73</td>
<td>1.53</td>
</tr>
<tr>
<td>1997-98</td>
<td>3.80</td>
<td>5.15</td>
</tr>
<tr>
<td>1998-99</td>
<td>2.93</td>
<td>5.57</td>
</tr>
<tr>
<td>1999-2001</td>
<td>2.55</td>
<td>3.67</td>
</tr>
<tr>
<td>2001-2002</td>
<td>3.71</td>
<td>5.48</td>
</tr>
<tr>
<td>2002-2003</td>
<td>3.24</td>
<td>8.13</td>
</tr>
<tr>
<td>2003-2004</td>
<td>2.54</td>
<td>0.65</td>
</tr>
</tbody>
</table>

1995-2004   | 3.05        | 4.66            | 1.61                       | 2.99               | 3.95               | 0.97                       |
| 1995-2001  | 3.05        | 4.46            | 1.41                       | 2.34               | 2.80               | 0.46                       |
| 2001-2004  | 3.17        | 5.09            | 1.92                       | 4.04               | 6.47               | 2.43                       |

Source: authors’ calculation

The impact of schooling on changes in inequality can be explained by two factors. One is changes in inequality of years of schooling and the other is changes in returns from schooling. As we have observed earlier, schooling has become more equal across the population in Brazil. This in turn has contributed to a reduction in inequality: the higher level of education, the greater earnings per hour. However, rates of return from education also change over time. In this context, we look at the returns
to each year of schooling in Brazil over 1995-2004. Figure A2 presents the trends in the returns from schooling over two periods, 1995-2001 and 2001-2004. The results show that educational returns have declined at all levels. It is evident that across educational levels, the curve of returns has an upward sloping in the first period but a downward sloping in the second period. This suggests that the gap in educational returns widened in the first period but narrowed in the second period. While the widening gap indicates an increasing inequality, the narrowing gap implies a fall in inequality. Therefore, a sharp decline in inequality over the 2001-04 period is mainly due to the gap in educational returns that has narrowed over the period between higher and lower levels.

Figure A1: Returns from schooling

![Figure A1: Returns from schooling](image-url)
Appendix 2: Shapely Decomposition to Explain Contributions of Income Components for Pro-Poor Growth

Suppose there are four income components which include:

- $X_{1t}$: Per capita labour income at year $t$
- $X_{2t}$: Per capita social security income at year $t$
- $X_{3t}$: Per capita cash transfers at year $t$
- $X_{4t}$: Per capita non-social income at year $t$

Total per capita income at year $t$ is thus the sum of the four individual income components. Thus we can write

$$X_t = X_{1t} + X_{2t} + X_{3t} + X_{4t}$$

Suppose $\log(x^*(X_t))$ is the logarithm of social welfare at year $t$ calculated on the basis of total per capita income $X_t$, which can be calculated from equation (14). Then the growth rate of social welfare at year $t$ is given by

$$\gamma_t^* = \log(x^*(X_t)) - \log(x^*(X_{t-1}))$$  \hspace{1cm} (A.1)

The Shapely decomposition can be used to calculate the contribution of each income component to the growth rate of social welfare of the total per capita income $X_t$ as

$$\gamma_t^* = \gamma_t^*(C_1) + \gamma_t^*(C_2) + \gamma_t^*(C_3) + \gamma_t^*(C_4)$$  \hspace{1cm} (A.2)

where $\gamma_t^*(C_i)$, where $i$ varies from 1 to 4, is the contribution of the $i$th income component to the growth rate of total welfare. Thus (A.1) is the proposed decomposition method which can be used to analyze the net contribution of each income component to the growth rate of welfare. This equation can also be utilized to analyze the contributions of each income component to growth in total inequality. Using the Shapely decomposition, we can write the net contribution of each income component to the growth rate of total welfare as follows:

$$\gamma_t^*(C_i) = \frac{6}{24} \left[ \log x^*(X_{1t} + X_{2t} + X_{3t} + X_{4t}) - \log x^*(X_{1t-1} + X_{2t} + X_{3t} + X_{4t}) \right]$$

$$+ \frac{2}{24} \left[ \log x^*(X_{1t} + X_{2t-1} + X_{3t} + X_{4t}) - \log x^*(X_{1t-1} + X_{2t-1} + X_{3t} + X_{4t}) \right]$$
$$\frac{2}{24} \left[ \log x^*(X_{t_1} + X_{t_2} + X_{3t-1} + X_{4t}) - \log x^*(X_{t_{l-1}} + X_{t_2} + X_{3t-1} + X_{4t}) \right]$$
$$+ \frac{2}{24} \left[ \log x^*(X_{t_1} + X_{t_2} + X_{3t} + X_{4t-1}) - \log x^*(X_{t_{l-1}} + X_{t_2} + X_{3t} + X_{4t-1}) \right]$$
$$+ \frac{2}{24} \left[ \log x^*(X_{t_1} + X_{2t-1} + X_{3t-1} + X_{4t}) - \log x^*(X_{t_{l-1}} + X_{2t-1} + X_{3t-1} + X_{4t}) \right]$$
$$+ \frac{2}{24} \left[ \log x^*(X_{t_1} + X_{2t-1} + X_{3t} + X_{4t-1}) - \log x^*(X_{t_{l-1}} + X_{2t-1} + X_{3t} + X_{4t-1}) \right]$$
$$+ \frac{2}{24} \left[ \log x^*(X_{t_1} + X_{2t-1} + X_{3t-1} + X_{4t-1}) - \log x^*(X_{t_{l-1}} + X_{2t-1} + X_{3t-1} + X_{4t-1}) \right]$$

$$\gamma^*_t(C_2) = \frac{6}{24} \left[ \log x^*(X_{t_{l-1}} + X_{t_2} + X_{3t} + X_{4}) - \log x^*(X_{t_1} + X_{t_2} + X_{3t} + X_{4}) \right]$$
$$+ \frac{2}{24} \left[ \log x^*(X_{t_{l-1}} + X_{t_2} + X_{3t} + X_{4}) - \log x^*(X_{t_1} + X_{t_2} + X_{3t} + X_{4}) \right]$$
$$+ \frac{2}{24} \left[ \log x^*(X_{t_{l-1}} + X_{t_2} + X_{3t} + X_{4}) - \log x^*(X_{t_1} + X_{t_2} + X_{3t} + X_{4}) \right]$$
$$+ \frac{2}{24} \left[ \log x^*(X_{t_{l-1}} + X_{t_2} + X_{3t} + X_{4}) - \log x^*(X_{t_1} + X_{t_2} + X_{3t} + X_{4}) \right]$$
$$+ \frac{2}{24} \left[ \log x^*(X_{t_{l-1}} + X_{t_2} + X_{3t} + X_{4}) - \log x^*(X_{t_1} + X_{t_2} + X_{3t} + X_{4}) \right]$$

$$\gamma^*_t(C_3) = \frac{6}{24} \left[ \log x^*(X_{t_{l-1}} + X_{t_2} + X_{3t} + X_{4}) - \log x^*(X_{t_1} + X_{t_2} + X_{3t} + X_{4}) \right]$$
$$+ \frac{2}{24} \left[ \log x^*(X_{t_{l-1}} + X_{t_2} + X_{3t} + X_{4}) - \log x^*(X_{t_1} + X_{t_2} + X_{3t} + X_{4}) \right]$$
$$+ \frac{2}{24} \left[ \log x^*(X_{t_{l-1}} + X_{t_2} + X_{3t} + X_{4}) - \log x^*(X_{t_1} + X_{t_2} + X_{3t} + X_{4}) \right]$$
$$+ \frac{2}{24} \left[ \log x^*(X_{t_{l-1}} + X_{t_2} + X_{3t} + X_{4}) - \log x^*(X_{t_1} + X_{t_2} + X_{3t} + X_{4}) \right]$$
Similarly, we can calculate the contribution of each income component to the growth rate of total per capita income:

\[
y_{t} = \gamma_{t}(C_{1}) + \gamma_{t}(C_{2}) + \gamma_{t}(C_{3}) + \gamma_{t}(C_{4}) \tag{A.3}
\]

Subtracting (A.3) from (A.2) gives the contribution of each income component to the inequality of total per capita income.

\[
g^{*}_{t} = g^{*}_{t}(C_{1}) + g^{*}_{t}(C_{2}) + g^{*}_{t}(C_{3}) + g^{*}_{t}(C_{4}) \tag{A.4}
\]