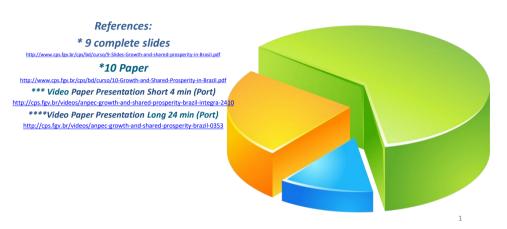
Growth and Shared Prosperity in Brazil

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A Simple Indicator of Shared Prosperity (Bottom 40%) = the 10th SDG

Following Atkinson, we can write a general continuous social welfare function denoted as:

$$W = \mu(x *) = \int_0^\infty u(x)w(x)f(x)dx$$
 This should satisfy: $\int_0^\infty w(x)f(x)dx = 1$

Suppose x is the income of an individual which is a random variable with density function f(x), then the mean income of the population is defined as:

$$\mu = \int_0^\infty x f(x) dx \tag{1}$$

The idea of shared prosperity (SP) is simply the mean of the bottom 40% of the population. More formally, suppose z is the income defined by:

$$0.4 = \int_0^z f(x) dx$$

then the Shared Prosperity (SP) indicator is defined by:

$$\mu_s = \frac{\int_0^z x f(x) dx}{\int_0^z f(x) dx}$$

which shows that the Shared Prosperity (SP) indicator is a weighted average of individual incomes. The SDGs focuses on the bottom 40% of the population. The idea is that a large proportion of the population should take part in and benefit from the growth process.

A Simple Indicator of Shared Prosperity

Applying the same decomposition the SP welfare indicator can be written as:

$$\mu_s = \mu(1 - I) \tag{3}$$

Where like other measures of social welfare defined over individual incomes such as Atkinson's (1970), it has an implicit (relative) inequality measure, defined as:

$$I = 1 - \frac{\mu_{\mathcal{S}}}{\mu} \tag{4}$$

which is a similar form of social welfare functions defined over the income space such as Atkinson's and Sen's indexes. Note that I is not a usual measure of inequality such as the Gini index because it does not satisfy the weak transfer axiom. It will be referred to as a measure of inequity in shared prosperity and (1-I) as a measure of equity in shared prosperity.

Or an Absolute measure of inequality:

$$A = \mu - \mu_s \tag{5}$$

Shared Growth

$$\mu_s = \mu(1 - I) \tag{3}$$

The idea of shared growth is now developed. To do so write (3) as:

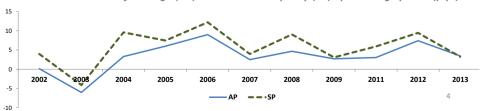
$$Ln(\mu_s) = Ln(\mu) + Ln(1-I)$$
 (6)

which on taking the first difference gives:

$$\gamma^* = \gamma + g \tag{7}$$

where the first term is the growth rate of shared prosperity, the second term is the growth rate of average prosperity and g is the growth rate of equity in shared prosperity, which will be positive (negative) if equity in shared prosperity is increasing (decreasing). Thus, there will be a gain (loss) in growth rate when equity is improving (deteriorating).

Annual Growth Rates of Average (AP) and Shared Prosperity (SP)- percentage points (p.p.)



Labor Deconstruction

Per Capita Labor Income in the total population can be expressed as:

Total Labor Earnings

Mean Earnings of those with Earnings

We can continue decomposing each peace of the identity in elements, what helps to understand the relative weight of each labor ingredient.



Definitions and Formulas Labor Economics

Occupied population (E): People working Unemployed population (U): People looking for a job but Economically Active Population EAP (PEA)

Inactive population (I): People not occupied Active Age Population AAP (PIA):

Total Population (POP): Active Age Population AAP (PIA) + People Out of Working Age (e.g. Children and the Elderly)

occupied + unemployed (E + U)

Participation Rate: (PEA)/(PIA) = (E + U)/(E + U + I)Unemployment Rate: (Unemployed)/(PEA) = (U)/(E+U)occupied + unemployed + inactive (in AAP) = (E + U + I) Occupation Rate in PEA: (Occupied)/(PEA) = (E)/(E + U)

Which labor ingredient contributed the most to growth in labor income?

2001 - 2013

Contribution of labor ingredientes to Growth in Total Labor Egrnings in annualized p.p

	Mean	10% Poorest	40% Poorest	Middle Group 40to90%	5% Richest
Employment Rate	0.25	-0.06	0.11	0.32	0.11
Participation Rate	0.38	-0.90	-0.19	0.61	0.79
Hours Worked	-0.39	-0.75	-0.44	-0.43	-0.21
Hourly Wages	2.93	5.59	5.60	3.07	1.64
Labor	3.18	3.88	5.08	3.57	2.33





Source: SAE from PNAD/IBGE microdata

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Hours Worked	-0.39	-0.75	-0.44	-0.43	-0.21
Years of Schooling	2.23	4.90	4.22	1.97	0.54
Premium per School Year	0.69	0.70	1.38	1.10	1.10
Labor	3.18	3.88	5.08	3.57	2.33

Source: SAE from PNAD/IBGE microdata

Definitions and Formulas

Labor Economics

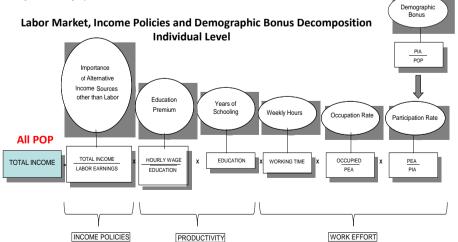
Occupied population (E): People working **Unemployed population** (U): People looking for a job but occupied + unemployed (E + U)not occupied

Inactive population (I): People not occupied Active Age Population AAP (PIA):

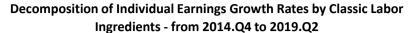
occupied + unemployed + inactive (in AAP) = (E + U + I)

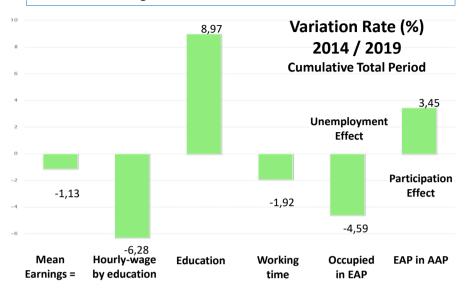
Total Population (POP): Active Age Population AAP (PIA) + People Out of Working Age (e.g. Children and the Elderly) Economically Active Population EAP (PEA)

Participation Rate: (PEA)/(PIA) = (E+U)/(E+U+I)Unemployment Rate: (Unemployed)/(PEA) = (U)/(E+U)Occupation Rate in PEA: (Occupied) / (PEA) = (E) / (E + U)



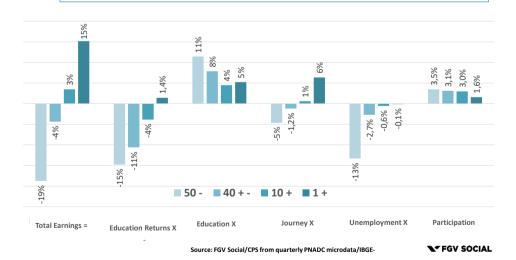
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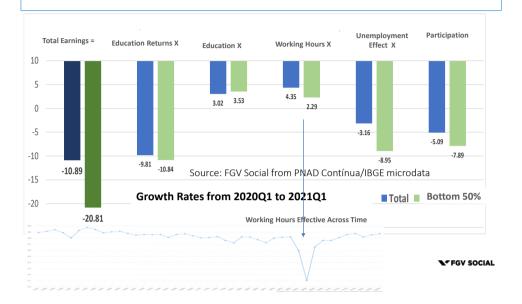
Source: FGV Social from PNADC/IBGE microdata individual normal Labor Earnings

Decomposition of Individual Earnings Growth Rates by Classic Labor Ingredients - per Capita Income Groups from 2014.Q4 to 2019.Q2



https://cps.fgv.br/en/inequality

Decomposition of Individual Earnings Growth Rates by Classic Labor Ingredients during the Pandemic % - per Capita Income Groups from 2020Q1 to 2021Q1



Income Sources as Determinants of Shared Growth Trends in Brazil

Suppose μ_t is the AP in year t and μ_{it} is the mean of the ith income component in year t.

$$\mu_t = \sum_{i=1}^k \mu_{it}$$
 (15)

Then it can be showed that:

$$\Delta Ln(\mu_t) \sim \frac{1}{2} \sum_{i=1}^{k} (\frac{\mu_{i(t-1)}}{\mu_{(t-1)}} + \frac{\mu_{it}}{\mu_t}) \Delta Ln(\mu_{it})$$
 (16)

which shows that the growth rate of AP is the weighted average of the growth rates of individual income components - the weights being proportional to the average of income shares in each period. This equation informs the magnitude of contribution of each income component to the growth rate of AP. Similarly, we can explain the contribution of each income component to growth rate of SP using:

$$\Delta Ln(\mu_{st}) \sim \frac{1}{2} \sum_{i=1}^{k} \left(\frac{\mu_{is(t-1)}}{\mu_{s(t-1)}} + \frac{\mu_{ist}}{\mu_{st}} \right) \Delta Ln(\mu_{it}) \tag{17}$$

where μ_{st} is the SP in year t and μ_{it} is the mean of the ith component of the bottom 40% of the population in year t. This equation informs the magnitude of the contribution of each income component to the growth rate of SP. Shared growth is defined as the gain/loss in the growth rate of the SP, which is the difference between the growth rates of SP and AP. The difference of growth in (16) from (17) provides the contributions of each income component to shared growth.

Determinants of Total Per Capita Income Distribution – By Income Sources Contribution to Annual Growth Rates By Segments 2004-14



Source: Author's calculation from PNAD/IBGE microdata

Social Security benefits identification can be decomposed in those above the Minimum Wage (MW) and those equal to 1 MW

Shared Opportunities

Suppose o(x) is an opportunity enjoyed by a person with income x, then the average opportunity (AO) enjoyed by the society is given by:

$$\vartheta = \int_0^\infty o(x) f(x) dx$$

This is the average opportunity available, but does not inform how it is shared by the population. Similar to the idea of shared prosperity we can define shared opportunity (SO) as:

$$\vartheta_s = \frac{\int_0^z o(x) f(x) dx}{\int_0^z f(x) dx}$$

which is the average opportunity enjoyed by the bottom 40% of the population. The inequity in opportunity can be defined as:

$$I_o = 1 - \frac{\vartheta_s}{\vartheta}$$

Then the SO can be written as:

$$\theta_s = \theta(1 - I_0)$$

which is similar to Atkinson's and Sen's social welfare functions but defined over opportunity space. I_o measures the proportional loss (or gain) in opportunity due to inequity (or equity) and therefore can be an indicator of inequity (equity) in opportunity. Note that unlike inequity measure I defined in (3), which lies in the range $0 \le I \le 1$, this inequity measure I_o lies in the range $-1 \le I_o \le 1$. The negative (positive) value implies that that opportunity is inequitable (equitable).

Occupation Opportunities

2001 - 2013

Average and Shared Opportunity in the Occupation Rate - %

Table 9: Average and shared opportunity in occupation rate					
Year	Average Opportunity	Shared Opportunity	Inequity		
2001	90.50	85.18	5.88		
2002	90.73	85.67	5.57		
2003	90.16	84.11	6.71		
2004	90.88	85.26	6.18		
2005	90.47	84.86	6.20		
2006	91.39	85.63	6.31		
2007	91.67	85.80	6.41		
2008	92.70	87.36	5.76		
2009	91.52	84.86	7.28		
2011	93.08	86.42	7.15		
2012	93.62	87.09	6.98		
2013	93.30	86.30	7.51		
Trend 2001-2013	0.28	0.15	0.12		

Source: Author's calculation from PNAD/IBGE microdata

Shared Opportunities

2001 - 2013

Trends in Average and Shared Opportunities in 2001-2013

Table 9: Trends in average and shared opportunities in 2001-2013				
Type of Opportunity	Average Opportunity	Shared Opportunity		
Occupation Opportunities (Employment Rate)	0.28	0.15		
Productive Employment (% with formal contract)	1.00	1.45		
Education Attainment (Years of Schooling)	0.17	0.21		
School Attendance (% 6-14 years old attending school)	0.26	0.39		

Source: Author's calculation from PNAD/IBGE microdata

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