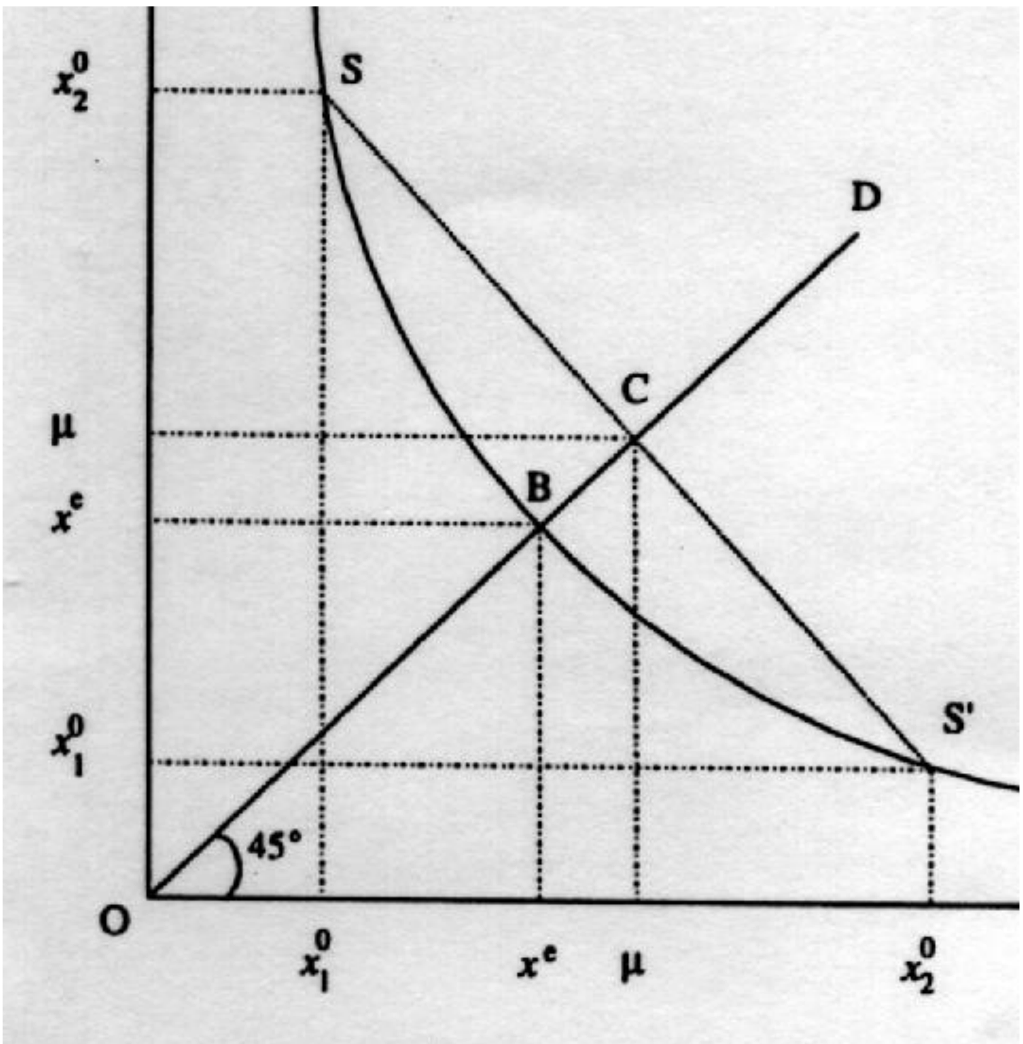
SOCIAL WELFARE *01.04

References:
Main reference text 01.05 Deaton (1997), Chapter 3, section 3.1 (section 3.2 will be used on the Poverty part). See also text 01.01
₴ Based on Atkinson's classic "On The Measurement of Inequality" (1970)

Social Welfare Function (SWF)

$$W = V(x_1, x_2, \ldots, x_N)$$



SWF function is a sum across individuals (typically of per capita expenditures or income) .

Properties of BES Functions

- Pareto Optimum – V is increasing (non decreasing) in its arguments. If one gets better and nobody worse it increases – to accommodate poverty measures (truncated BES functions) we adopt non decreasing function.
- Symmetry or Anonymity – BES depends on individual welfare levels and not their identity.
- Principle of Transfers (Pigou – Dalton) – For a given total X, BES function will be at it maximum point when inequality will be at the same time at its minimum, conditioned to the average (when OC' S are equal) – express an equity preference. Ignore any kind of restrictions on allocations and incentives effects.
  Decreasing marginal utility (quasi-concavity or more general S – concavity) . If $\mathbf{x}_1$ and $\mathbf{x}_2$ are lists of x's and if V ($\mathbf{x}_1$) =V ($\mathbf{x}_2$) then $\lambda \mathbf{x}_1 + (1 - \lambda) \mathbf{x}_2$ for a λ – [0 , 1] will have a higher value or equal to the original allocations.

SOCIAL WELFARE AND INEQUALITY

If V is homogeneous of the 1st degree

$$W = \mu V\left(\frac{x_1}{\mu}, \ldots, \frac{x_N}{\mu}\right)$$

Separate inequality and mean effects

If we normalize units as V (1,1,...,1) = 1
When there is perfect equality, that is, everybody have individual level of welfare, social welfare has the same value.

$$W = \mu (1 - I)$$

By the transfers principle, inequality is the cost that makes the value of social welfare falls below the perfect equality point.

$X^e$is the equivalent of x equally distributed
BC/OC is the geometric measure of inequality proposed by Atkinson.
One advantage of this approach is to differentiate inequality and social welfare. It is consistent with poverty.

From: Social Welfare Function **To: Inequality**

$$W = \frac{1}{N}\sum_{i=1}^{N}\frac{x_i^{1-\epsilon}}{1-\epsilon}, \quad \epsilon \neq 1$$

$$I = 1 - \left(\frac{1}{N}\sum_{i=1}^{N}(x_i/\mu)^{1-\epsilon}\right)^{1/(1-\epsilon)}$$

Welfare with inequality aversion $\in \geq 0$ Controls the degree of aversion towards inequality – Figure above when $\in$ is smaller the flatter is the curve .

Marginal Rate of Social Substitution

$$\frac{\partial W/\partial x_i}{\partial W/\partial x_j} = (x_j/x_i)^{\epsilon}.$$

If $\in = 0$, then marginal utility is fixed and I do not take inequality into account.
If $\in = 2$ and $x_i = 2xj$ then marginal social utility of giving x to i is 1/4 (of giving x to j) .If $\in = -\infty$, then utility is similar to Leontief type (Raws) , that is, what matters is the welfare of the poorest individual of society.
See step by step Derivation of inequality from the Social Welfare Function in 01.04.

Atkinson measure (For $\varepsilon = 1$)

From: Social Welfare Function To: Inequality

$$W = \left(\prod_{i=1}^{N} x_i\right)^{\frac{1}{N}}$$

$$I = 1 - \left(\prod_{i=1}^{N} x_i\right)^{\frac{1}{N}}$$

OTHER INEQUALITY MEASURES

Gini Index – It can be derived directly from social welfare function with weight structure equal to (1- F(x)) of individual incomes, see above) reaching aggregated $\mu (1 - \delta)$ where μ is the average income, δ the Gini coefficient and ρ ~ [0 , 1] from Sen (1976). Or more generally, $\mu (1 - \delta)^{\rho}$ where ρ is the inequality aversion parameter from Graff (1981). Gini is popular due to its tradition, scale and intuition. Disadvantages: Does not change much, low sensibility to bottom income changes and not very adapted to decompositions.

Bottom 40% Share in total income – **Shared Prosperity** as in Goal 10 of Sustainable Development Goals (SDGs). Sensitive by construction to the lower end of income distribution. Derived directly from Social Welfare function but does not follow the principle of transfers.

Interquartilic Amplitude : (statistical approach, less used in economics).

$\frac{\text{Income 75\%}}{\text{Income 25\%}}$ Do not follow the principle of transfers. Transfering from a low quartile for someone poorer can raise inequality.

Total amplitude

$\beta_1 = \frac{[\text{Max } y_i - \text{Min } y_i]}{\mu}$ or $\beta_2 = \frac{\text{Max } y_i}{\text{Min } y_i}$ disadvantage: very sensitive to *outliers*

Palma Ratio*:* It is the ratio of the richest 10% of the population's share of income divided by the poorest 40%'s share. Recent but already popular.

Variance of Logs $V \log = \frac{1}{N} \sum (\overline{\log y} - \log y_i)$

Advantages: Insensitive to scale, Allows disaggregation

Disadvantages: Do not exist for yi=0, Little sensitive on the top, - Does not follow the transference principle (critic less relevant in practice for inequality, more for concentration measures). Decompositions works out nicely in a log-linear regression framework

Coefficient of Variation

$$CV = \frac{1}{\mu}[\frac{1}{N} \sum (y_i - \mu)^2]^{1/2}$$

Theil T and Theil L and General Entropy Indexes

– Belong to the family of entropy measures (other than social welfare function deduction approach). Highly decomposable

$$T = \frac{1}{N} \sum_{i=1}^{N} \frac{x_i}{\mu} \ln\left(\frac{x_i}{\mu}\right) \qquad \text{e} \qquad L = -\frac{1}{N} \sum_{i=1}^{N} \ln\left(\frac{x_i}{\mu}\right),$$

Where xi is individual ***i income****,* ***N is*** population size and μ is mean income.

C. General Entropy S- measure nests Theil T and Theil L as special cases.

$$S = \frac{1}{\varepsilon(1-\varepsilon)}\left[1 - \frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i}{\mu}\right)^{1-\varepsilon}\right]$$

OBS: **ε=0 Theil T; ε=1 Theil L; ε= -1 Coefficient of Variation**