Designing a System of Social Targets

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Abstract: **This paper discusses the economic rationality of a system of social targets, as a way for the federal government to increase efficiency in the use of its social budget transferred to municipalities. The paper develops extensions of a standard principal-agent framework in various directions including static models with and without imperfect information and dynamic models with complete and incomplete contracts.**

The results of the static models show that the use of the focalization criteria where the poorest municipalities get more resources may lead to adverse incentives to poverty eradication. We also show that unconditional transfers from the federal government totally crowd-out local social expenditures. The paper argues in favor of the use of contracts where the greater the improvement in relevant social indicators, the more resources each municipality would receive. The introduction of imperfect information basically generates a penalty to the poor segments in areas where local governments are less averse to poverty.

An advantage of this type of contract is also to reduce the problem of political favoritism when certain social groups receive greater, or smaller, attention from specific governments. With the establishment of social targets it becomes possible to generate proper incentives so that social spending is distributed more equitably between groups.

Key words:
1. social targets
2. poverty
3. inequality
4. social spending
5. social welfare

Summary

Designing a System of Social Targets

1 – Introduction

Starting in the nineties, an increasing number of countries began to implement monetary policies characterized by the establishment of inflation targets. With the 1999 exchange rate crisis and the shift to a floating exchange rate, Brazil also adopted this system. As a consequence of this new policy, the instruments of monetary and exchange rate policy have been used by the Central Bank to lower inflation in accordance to pre-established targets, widely informed to society.

In reference to the system of inflation targets, there exists a vast and well-known literature on the theme, which served as a starting point for the discussions about its implementation. However, when we talk about social targets, the discussion tends to be politicized, and the economic aspects of the matter are barely discussed. In this text we wish to rescue the economic discussion, showing in which way does the implementation of a system of social targets can bring efficiency gains in the use of public money.

The matter of efficiency in the use of public money is essential in a country as Brazil. Brazil has a significant proportion of its GNP allocated in social spending —around 21% of GNP which is the highest in Latin America[1]. Nonetheless, the country displays lousy social indicators and an embarrassing distribution of income, especially when compared to other countries with similar levels of income per capita.

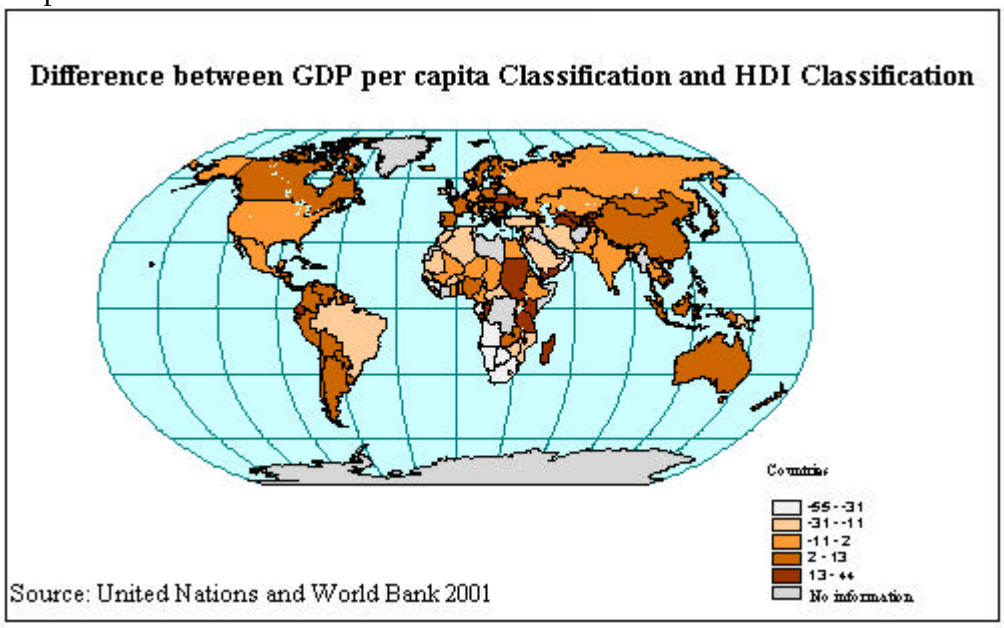
In graph 1, countries classified in accordance to the difference between their rankings in terms of income per capita and Human Development Index (HDI) levels . Brazil position in terms of income per capita levels is superior superior to the great majority of nations but not its Human Development Index (HDI) ranking. This means that the development in Brazilian living conditions did not accompany the extent of its economic development.

Given the share of GDP spent in the social area and per capita income level, Brazil problem does not seem to be the magnitude of resources devoted to the social area, but the efficiency of its use. The creation of mechanisms to monitor the social budget's quotas is important, not to verify if they are really being employed in the pre-determined areas (education, health, etc), but mainly to check their effective impact on living conditions. It is not enough to know how much money was employed in certain areas; it is necessary to measure results.

Many social programs are based upon the transfer of federal government's funds to the poorer regions. Obviously, the expenditure of money in these regions results in an improvement for the local population's living conditions. However, what is not being evaluated—and what establishes the core of this work—is to know whether the final result reached could have been better.

[1] See Neri et all (1999).

Graph 1



What we will analyze, as follows, are social target mechanisms based on the classic principal-agent framework. The principal may be regarded as the federal government, for example, who wishes to improve the living conditions of the poorest population by transferring funds to municipalities. An example of a program of this type is the Alvorada Project, through which the federal government has implemented various programs in the realms of education, health and income, after identifying the micro-regions in the country with the worst HDI, and thus aiming towards their improvement[2].

In light of Brazil's size and complexity, it is impossible for the federal government to know which are the specific needs of each locality within the country. In a region where the HDI struck as low, it would rarely have more information than the local government about who are the poor and what is the best way to help them, for the mayor is the one who better understands the region's intricacies. For this reason, it is only natural for the local government to be responsible for determining what must be done. The federal government should have the assignment of establishing a partnership with municipalities, via target contracts, and monitor how funds are being spent and which are the goals being achieved.

Usually, the government limits itself to performing an analysis regarding the legality of the fund's use. An even more important analysis—to measure the social result attained by the transferred funds—usually tends to not be performed. What is analyzed, in general, if whether the funds were employed in accordance to the law, but not social results achieved.

Facing this situation, we analyze the mechanisms for social targets in relation to the fulfillment of targets by the ones receiving the funds, as pre-established in contract. The

[2] The HDI is an index composed of health, education and income indicators, being that each one of these three components has the same weight in the index.

mechanisms being analyzed are based upon the selection of an optimum level of governmental transfers—for example, from the federal government to municipalities.

In the studied system of social targets, it is the government's responsibility to establish a group of possible contracts to be asserted between the federal government and the municipality. Such contracts contain clauses to establish the targets to be reached and the value to be forwarded from the federal government to the local one for the accomplishment of these goals. The subjacent idea is that, if the municipality does not reach the established targets, it will not receive the funds, or receive only an amount proportional to the accomplishment reached. This way, what is established between the federal and local governments is similar to a hiring contract, in which the federal government hires the municipality so that it may run a service in the social area. However, in a more realistic situation, so that the targets may be reached, first the municipality must receive the funds, and only after the targets are checked. We can consider the funds received by the local government as an advanced payment – called here as Social Credit - so that the municipality may carry out a specific service pre-determined in the contract—which establishes the goals to be accomplished. In case the targets are not reached, the municipality starts to have a debt with the federal government for the non-fulfillment of an agreed service. The debt is the difference between the advanced payment and payment estimated by the contract for the complete results to be accomplished.

The main issue in this type of model is the establishment of the targets to be reached and the manner of paying for the obtained result.

2 – Basic Model

The model is based on the structure of principal and agent. In our case, the federal government (F) may be regarded as the principal. The agents are the municipal governments (M), here forth referred to as municipalities. Aside from the federal and municipal governments, we have the poor (P), whom the social targets to be established in contract between the government and the municipality will be affecting.

A basic hypothesis of the model is that the federal and local governments seek to improve living conditions of the poor, for this means to the representatives an increase in their chances of reelection. In the model, their level of income will measure this improvement in living conditions of the poor. This is equivalent to saying that the social target sought is the increase of average income of the poor.[3]

However, the key issue when discussing poverty reduction, is to know who will pay the bill. If on one hand, the reduction of poverty brings electoral benefits, on the other hand, for it to occur, it is necessary to invest in income transfer programs, which reduces the available budget for other types of investments.

The local government would love it if the federal government made large social investments in its region, and preferably, if such expenses did not include a counter-measure from the municipality. It would be the authentic "free lunch." The federal government would spend part of its budget, and the municipality would obtain political gains. The same analysis is valid in the opposite sense.

Such as Besley (1997), Gelbach and Pritchett (1997), and Azam and Laffont (2001), we assume that the federal government, as well as the local one, has an aversion to poverty, which may be modeled through a utility function, in which the poor's income is seen as a positive externality for the federal government as well as for the local government. For a matter of simplicity, we assume that the government's and the municipality' utility functions are quasi-linear, in the available budget, and strictly concave in the poor's income. This way, the government and the municipality are concerned with absolute poverty, instead of relative poverty. The desire to help the poor does not depend, however, on the total budget, but only on the poor's income level.

The utility functions for the federal government , U_F, and for the municipality, U_M, are respectively given by:

$U_F = G_F + N_{P.}\, v(Y_P)$
$U_M = G_M + N_P.?.v(Y_P)$

[3] However, an identical analysis can be made with other social indicators or even with an average of them, such as occurs with Human Development Index—HDI—or with the Life Conditions Index—LCI (Índice de Condições de Vida—ICV). Where one reads *income*, child mortality, school attendance rate, HDI, etc. could be placed instead. The choice of the target *income* throughout the text has the objective of trying to make the model more intuitive.

Being that $v(0) = 0$, $v'(Y_P) > 0$, $v''(Y_P) < 0$, $\lim_{Yp? \ 0} v'(Y_P) = +?$ e $\lim_{Yp? \ +?} v'(Y_P) = 0$

Where,

G_F: is the budget available to the federal government. It is considered that the government has a total budget (own) of Y_F. Part of this budget may be transferred, T, to income programs directed towards the poor. The difference $Y_F - T = G_F$. This is the budget the government has for all other necessary expenses. Obviously, the greater the available budget, the larger will be the government's utility.

G_M: budget available to the municipality. Such as the government, the municipality also has its own budget, Y_M. The available budget, G_M, is what is left after the transfer performed by the municipality to the poor.

?: is the parameter expressing the aversion to poverty of a local government. Different mayors may present different degrees of aversion to poverty. The absence of the parameter ? in the government's utility function expresses the normalization that it has a parameter of ? = 1.

N_P: number of poors in a municipality.

We will assume that the local government is the one better aware of the local reality, and therefore more capable than the federal government of identifying who really are the poor within the region. The local government also has better conditions for managing and implementing an income transfer program to its locality. This way, all government transfers will be directly made to the municipality, which will be responsible for transferring it to the poor.

In relation to the poor's utility, U_P, the only consideration undertaken by us will be that if grows in accordance to income: $U_P'(Y_P) ? 0$. The greater the income, the poor will be better off.

From here on we will sometimes refer to the federal government as the *principal* and to the local government as the *agent.*

3 – Static Model

In this chapter, we divide the analysis in two parts. One refers to the case of complete information, when the principal knows the type ? of the agent. In the other case, there is an information asymmetry, derived from the non-observance type of agent. This asymmetry allows for some agents to attain informational income, which can be seen as a counterpart that the agent charges to reveal its true type.

3.1 – Complete Information

In this case, the government knows the mayor's (municipality's) aversion to poverty. It is an ideal situation, as it is difficult to know this type of information. However, the study in this case is important for some reasons. One of them, is that it allows us to compare the differences in the results of social policies when the government does not know the type of municipality. Besides this, we can obtain some interesting intuitions, which are the key factors in determining the result of social policies.

3.1.1 – Autarchy (A)

The basic situation is that in which the government does not carry out any transfer to the municipality. In this case, the municipality's incentive to transfer income to the poor is exclusively due to the positive externality that an improvement in the poor's living conditions results to the local government. In this situation, the municipality solves the following problem:

$$\underset{Y_P}{\text{Max}}\ G_M + N_P .? \ . \ v(Y_P)$$
$$\text{s.t: } G_M + N_P \ . \ Y_P \ ? \ \ Y_M$$

The first order condition (FOC) of the above problem is:

$$v'(Y_P^A) \ ? \ \frac{1}{?} \qquad \log o$$

$$?_1 \ ? \ ?_2 \ ? \ \ Y_{P_1} \ ? \ Y_{P_2}$$

However, the poor's income in autarchy, Y_P^A, is determined by the coefficient of the local government's aversion to poverty. The larger this coefficient, the larger will be the poor's income. Governments more concerned with the poor's social situation implement better income transfer policies. It is observed that the poor's income does not depend upon the number of poors nor on the municipality's budget. This is a result of the quasi-linear utility function chosen for the local government.

For the municipality of type ?, the utility after the transfer is:

$$U(?) ? U_M^A ? Y_M ? N_P.Y_P^A ? N_P.?.v(Y_P^A)$$

Further ahead, when we deal with the federal-local relation, this equation will be the minimum utility that the municipality will take into consideration to accept the establishment of a contract estimating social targets as a countermeasure to the governmental transfers.

3.1.2 – Unconditional Transfer (T^I)

Suppose the federal government chooses to invest in determined places, transferring funds for the municipality to invest in a social area. As we have previously calculated, in our model we will always suppose that the government transfers funds to the municipality and the local government is the one in charge of implementing the social policies. In this case, let's suppose the government does not establish any condition (i.e., social target) in what refers to the accomplishment of results by the municipality. It only transfers unconditionally a fixed fund of T^I. For the municipality, the problem to be solved is:

$$\begin{array}{l} \text{Max } G_M ? N_P.?.v(Y_P) \\ Y_P \\ \text{s.a}: \ G_M ? N_P.Y_P ? Y_M ? T^I \end{array}$$

Solving the problem, the first order condition obtained is:

$$v'(Y_P^I) ? \frac{1}{?} \quad ? \quad Y_P^I ? Y_P^A$$

That is, the poor's income in autarchy or in a situation in which an unconditional transfer occurs is the same.

Proposition 1: If the federal government performs unconditional transfers to the local governments, the poor's situation does not change.

Besides this,

$$U_M^I ? Y_M ? T^I ? N_P.Y_P^I ? N_P.?.v(Y_P^I) \overset{Y_P^I ? Y_P^A}{?} Y_M ? T^I ? N_P.Y_P^A ? N_P.?.v(Y_P^A)$$

$$U_M^I ? U_M^A ? T^I \quad ? \quad U_M^I ? U_M^A$$

and

$$U_F^I ? U_F^A ? T^I \quad ? \quad U_F^I ? U_F^A$$

Defining the funds destined, by the municipality, to the social program as being T_M, we have that:

$T_M^I \ ? \ N_P.Y_P^I \ ? \ N_P.Y_P^A \ ? \ T_M^A$

What is observed in this type of transfer is that the local government does not use the funds transferred to improve the poor's situation, but starts to include it in its available budget. Another interpretation is to consider that the local government really destines the funds received to the social programs. However, in the same quantity as that received, it stops directing part of its own budget to the social area, accounting for these funds as available budget. It would be a type of crowding-out effect, where the government's investment reduces (misplaces) the municipality's own investments.

In this way, the local government's utility increases, for the poor will be as well off as they would in autarchy, but the available budget increases. The government, on the other hand, will be worse off, for the poor will not have improved, and the available budget will be smaller.

3.1.3 – Perverse Incentive (PI)

Suppose the government decides to help more the municipalities where the poor are poorer, so that the smaller the poor's income, the greater is the income per capita transfer carried out by the government to the municipality. For this, we suppose the government transfers the difference between Y_P, and a basic estimated value, K. Soon, the total transfer that a municipality is entitled to is:

$T \ ? \ (K \ ? \ Y_P).N_P$

The municipality, knowing that it will be entitled to this transfer, solves the problem of determining how much it will invest in the social area, that is, what is the income $N_P.Y_P$ that it will transfer to the poor. The better the poor's situation, the less the municipality will received from the government, but on the other hand, the greater is the externality created by the poor's situation. The municipality's problem can be described as:

$\text{Max } G_M + N_P .? \ . \ v(Y_P)$
Y_P
$\text{s.t: } G_M + N_P \ . \ Y_P \ ? \ Y_{M+}(K - Y_P).N_P$

Solving for this, we have:

$v'(Y_P^{IA}) \ ? \ \frac{2}{?}$ such that,

$Y_P^{IA} \ ? \ Y_P^A$

The consequence of establishing a system in which the greater the poverty, the greater the federal government's investment in the region, without any counter-measure regarding the

results, is the creation of perverse incentives. This is due to the fact that it stimulates the municipal government to reduce its social investments, so that it can receive more transfers. The final investment ends up being smaller than in the case of autarchy.

3.1.4 – Transfer Conditional on the Fulfillment of Social Targets (T^c)

Until now we have studied cases in which the government either undertook no transfers of any kind to social programs, or it did so without establishing any type of social target that could serve as a condition for the municipality to receive funds. Let's now study how the establishment of social targets can increase efficiency in the use of public money.

Let's suppose that the principal offers a contract to the agent under which a transfer conditioned upon the achievement of a pre-determined income social target, Y_p is estimated. The principal's problem is defining a contract. $(T^C(?), Y_P(?))$, under which the agreement with the agent's type ? is established in its target, Y_P, and the transfer, T_C, corresponds to the target's accomplishment. For this, it is necessary to guarantee that, in accepting the contract, the agent will obtain at least the same utility it would have in autarchy—this is the well-known Restriction of Participation (RP). This way, the principal's problem is:

$$\begin{array}{ll} \text{Max} & Y_F ? T^C(Y_P) ? N_P.v(Y_P) \\ \{Y_P, T^C\} & \\ \text{s.a:} & (Y_M ? T^C(Y_P) ? N_P.Y_P) ? N_P.?.v(Y_P) ? U(?) \quad (RP) \end{array}$$

From RP we have that:

$$T^C(Y_P) ? U(?) ? Y_M ? N_P.Y_P ? N_P.?.v(Y_P)$$

Soon, the government's problem can be described as:

$$\begin{array}{l} \text{Max } Y_F ? (U(?) ? Y_M ? N_P.Y_P ? N_P.?.v(Y_P)) ? N_P.v(Y_P) \\ \{Y_P\} \end{array}$$

A first order condition is that:

$$v'(Y_P^C) ? \frac{1}{1 ? ?} \quad ? \quad Y_P^C ? Y_P^A$$

That is, with the transfer of funds from the federal government to the municipality being conditioned to the attainment of a specific social target—in our case the target being an increase in the poor's income—we see that the final income of the poor is greater than it would have been had there not been the establishment of targets. Without these, we see that the municipality ends up investing the same value with or without the government's transfer in the social area. All transfers made the increase in the available budget for the municipality's expenses in activities other than in the social realm redundant, although the government would have liked to witness an increase in these. The government would

transfer resources for the municipality to use in the social area, and the municipality would decrease in the equivalent proportion its own resources for that area. With the establishment of targets, this seizes to happen.

Proposition 2: the establishment of social targets increases the efficiency in the use of public money transferred to municipalities so that they can employ it in the social area, providing the attainment of social results better than without targets.

Aside from this, in relation to the funds directed from the municipality to the social area, we have that:

$$U_M^{TC} ? U_M^A$$

$$? \quad G_M^{TC} ? N_P.?.v(Y_P^{TC}) ? G_M^A ? N_P.?.v(Y_P^A)$$

$$? \quad G_M^{TC} ? G_M^A ? N_P.?.[v(Y_P^{TC}) ? v(Y_P^A)]$$

$$? \quad G_M^{TC} ? G_M^A$$

Therefore, when a contract is made with social targets, the municipality, aside from directing the resources received from government to the social area, it also increases the volume of resources that normally it would spend if there had not been any type of contract with the government. It is important to observe that when there weren't any targets, if the government had transferred T resources to the municipality, it would have decreased by T amount its own resources in the social area. Now, aside from not reducing any, it also increases the quantity of its own resources to invested in the social area.

If on the one hand, the municipality loses utility from having less available funds to its "non-social" expenses, in return it gains from the externality of improvement in the poorest's well-being, proportional to the investment made with the federal and municipal funds. Adam and O'Connell (1999) also found this type of result, in which the budget destined to the poor by the agent is greater than the funds received from the principal.

It is possible to state that a contract with social targets is capable of raising social investments. While in the contract with no targets, the volume of resources reaching the poor was the same with or without transfers, in this case, the one reaching the poor is greater than the sum of the government transferred funds and those desired by the municipality in conditions without the establishment of targets.

Impact of social targets: based on the CPO, it is possible to have an intuition about the degree of improvement that the social targets can bring on the poor's income. Let's remember that in the definition of our model, we normalized the government's aversion to poverty as being equal to one ($?_F$ =1). As a result of this, in the equation $v'(Y_P^{TC}) ? 1/(1 ? ?)$, the number 1 in the denominator is the government's $?_F$. If we had written the government's utility function as $U_F = G_F + N_P. ?_F. v(Y_P)$, we would have found as a first order condition:

$v'(Y_P^{TC}) ? \frac{1}{?_F ? ?}$, where ? , is the local aversion to poverty.

Linear Contract

A way of inducing the municipality of reaching the projected targets is to offer a contract of the type:

$T(Y_P) ? a ? b.Y_P$

In this contract, the municipality has a guaranteed fixed value. It is worth observing that this value may be positive as well as negative, implying in this last case that there is a penalty to be paid by the municipality in case the social results are very low. We also have a variable part. The higher the reached income, the greater the transfer. The coefficient "b," establishing the value of the variable part, is know for having an incentive power, for the greater its value, the greater is the municipality's incentive to reach even higher social results.

Proposition 3: The coefficients belonging to a linear contract of social targets are:

$a ? T(Y_P^{TC}) ? b.Y_P^{TC}$, onde $T(Y_P^{TC}) ? N_P.[(Y_P^{TC} ? Y_P^A) ? ?.(v(Y_P^{TC}) ? v(Y_P^A))]$

$b ? \frac{1}{1 ? ?}$

For proof, refer to Appendix I.

3.1.5 – Favoritism without Transfer (F)

Until now, we have considered that the local government had an aversion to poverty coefficient equal to that of all N_p poor. However, there commonly exists a preference for certain types, in detriment of others.

Empirical studies have shown that a large portion of poverty is spread among children and teenagers. 45% of the extreme poor in Brazil have 15 years or less of age against 30% of their share in the whole population, similar discrepancies are observed worldwide. Neri and Costa (2001) argue that the age distribution of poverty may be influenced by the fact that the youngest are not allowed to vote. In other words, the fact the youth is underrepresented in the electoral market makes social expenses on children less palatable to politicians. It is not a coincidence that family of many children and often headed by one female would be less subject to social spending. In modern democracies, the rule that each individual gets one vote does not apply, the rule is one adult, one vote [4] [5]..

[4] Another explanation for the preference of some poor individuals is the matter of electoral region. Many politicians know they have a greater acceptance rate in a region rather than the rest, and thus they prefer to favor the place where it is easier to attain votes and support. The same occurs in relation to certain professional categories, which tend to be preferred by some politicians.

Our objective is to model this type of political favoritism in relation to the determined group and comprehend in which form it impacts the distribution of resources driven towards the social area. In the future, we will show that the manner of establishing social targets can be of use to diminish the problem.

Let's make the assumption that there are two types of poor, whose populations are N_{P1} and N_{P2} for which the municipality's aversion to poverty coefficients are $?_1$ and $?_2$, respectively.

Not having any type of transfer coming from the government, the municipality's problem can be described as:

$$\underset{\{Y_{P1},Y_{P2}\}}{\text{Max}} \quad G_M + N_{P1} \,.\, ?_1 \,.\, v\,(Y_{P1}) + N_{P2} \,.\, ?_2 \,.\, v\,(Y_{P2})$$

$$\text{s.t:} \quad G_M + N_{P1} \,.\, Y_{P1} + N_{P2} \,.\, Y_{P2} \;?\; Y_M$$

The first order conditions are:

$$v'(Y_{P1}^F) \;?\; \frac{1}{?_1} \quad \text{e} \quad v'(Y_{P2}^F) \;?\; \frac{1}{?_2}$$

Supposing the poor of type $?_1$ are preferred, that is, $?_1 > ?_2$, we have $Y_{P1} > Y_{P2}$. That is, the preferred group receives an aid greater than the surpassed group.

3.1.6 – Favoritism Conditional on the Fulfillment of Social Targets (FC)

Let us now suppose that the main government does not have a preference for either types of poor in a determined municipality, and that it is willing to establish with the municipality a contract estimating a transfer of resources, T^{FC}, linked to the attainment of certain results in the social realm. In this case, the government's problem is:

$$\underset{\{Y_{P1},Y_{P2}\}}{\text{Max}} \quad G_F \;?\; N_{P1}.v(Y_{P1}) \;?\; N_{P2}.v(Y_{P2})$$

$$\text{s.a :} \quad G_F \;?\; T^{FC} \;?\; Y_F$$

$$G_M \;?\; T^{FC} \;?\; N_{P1}.?_1.v(Y_{P1}) \;?\; N_{P2}.?_2.v(Y_{P2}) \;?\; U_M^F \quad \text{(RP)}$$

The first order conditions are:

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[5] More generally, the sub-representation of the poor in electoral terms, would explain why fiscal spending frequently does not favor the poorest.

$$v'(Y_{P1}^{FC}) ? \frac{1}{1 ? ?_1}$$

$$v'(Y_{P2}^{FC}) ? \frac{1}{1 ? ?_2}$$

Where we conclude that:

$$Y_{P1}^{FC} ? Y_{P1}^{F}$$
$$e$$
$$Y_{P2}^{FC} ? Y_{P2}^{F}$$

Again, the use of a contract between the government and municipality, linking the resource transfer to the accomplishment of social targets, causes a result better than that attained without the targets. This improvement in the poor's living conditions occurs for both types of poor.

However, when we compare the solution attained when we had favoritism without the existence of a contract with social targets to the situation in which there are targets, we can verify that if type $?_2$ is favored for the local administration, we have that:

$$\frac{v'(Y_{P1}^{F})}{v'(Y_{P2}^{F})} ? \frac{1/?_1}{1/?_2} ? \frac{?_2}{?_1} ? \frac{1 ? ?_2}{1 ? ?_1} ? \frac{1/(1 ? ?_1)}{1/(1 ? ?_2)} ? \frac{v'(Y_{P1}^{FC})}{v'(Y_{P2}^{FC})}$$

Proposition 4: a contract with social targets would reduce the social difference among the group less favored and the group more favored by the municipality's social policies.

Observe the simple establishment of a contact with social targets does not guarantee that the differences between the groups are eliminated, although they serve to soften the discrimination problem felt by a specific group of poor. Eventually, for the two groups to have the same results, it would be necessary for the government to consider in its utility function the groups of poor in differentiated manners, given priority to those left behind by the municipality.

3.2 – Incomplete Information

The model with complete information is useful as a reference parameter, as it describes the optimum solution to the problem (first-best). However, so that we have a model portraying reality, it is interesting to soften some hypotheses. We now deal with the case where the type of agent is private information, such that it is unknown to the principal. This is equivalent to saying that the federal government does not know what is the local government's aversion to poverty, knowing only that historically there exists a specific distribution of types, with a certain probability of the municipality being the type more or less concerned with the social issue.

We will analyze two types of cases: in one of them, we will work with the existence of only two types of agents. In the other case, we will analyze what happens when we have infinity of types, distributed according to a density function.

3.2.1 – Two Types of Agents

Suppose that $\theta \in \{\underline{\theta}, \overline{\theta}\}$ and that the probability of the municipality being a type $\overline{\theta}$ is π. For the municipality to accept a contract establishing targets to be accomplished, the contract must guarantee at least the same utility obtained without it. This is the Participation Restriction (RP).

As is traditional in problems of adverse selection, the principal must offer a menu of contracts, that is, a contract for each type of agent. The contracts must also have been chosen in a way that the agent of a specific type does not try to pretend to be another type. This is the Incentive Compatibility Restriction (RCI).

The principal solve the following problem:

$$\underset{\{\overline{Y}_P, \overline{T}, \underline{Y}_P, \underline{T}\}}{\text{Max}} \; \pi.[Y_F - \overline{T} + N_P.v(\overline{Y}_P)] + (1-\pi).[Y_F - \underline{T} + N_P.v(\underline{Y}_P)] \qquad \text{(I)}$$

$$\text{s.a}: (Y_M - \underline{T} + N_P.\underline{Y}_P) + N_P.\underline{\theta}.v(\underline{Y}_P) \geq U^A \qquad (RP\,\underline{\theta})$$

$$(Y_M - \overline{T} + N_P.\overline{Y}_P) + N_P.\overline{\theta}.v(\overline{Y}_P) \geq (Y_M - \underline{T} + N_P.\underline{Y}_P) + N_P.\overline{\theta}.v(\underline{Y}_P) \qquad (RCI\,\overline{\theta})$$

As is commonly done, we consider the participation restriction of type $\underline{\theta}$ and the incentive compatibility restriction of type $\overline{\theta}$ to be active.

$$(RP\,\underline{\theta}): \qquad \underline{T} = U^A - Y_M + N_P.\underline{Y}_P + N_P.\underline{\theta}.v(\underline{Y}_P) \qquad (*)$$

$$(*) \text{ in } (RCI\,\overline{\theta}): \qquad \overline{T} = (U^A - Y_M) + N_P.v(\underline{Y}_P).[\overline{\theta} - \underline{\theta}] + N_P.\overline{Y}_P + N_P.\overline{\theta}.v(\overline{Y}_P) \qquad (**)$$

Substituting (*) and (**) in (I) we have:

$$\underset{\{\overline{Y}_P, \underline{Y}_P\}}{\text{Max}} \; \pi.[Y_F - [(U^A - Y_M) + N_P.v(\underline{Y}_P).[\overline{\theta} - \underline{\theta}] + N_P.\overline{Y}_P + N_P.\overline{\theta}.v(\overline{Y}_P)] + N_P.v(\overline{Y}_P)]$$

$$+ (1-\pi).[Y_F - [U^A - Y_M + N_P.\underline{Y}_P + N_P.\underline{\theta}.v(\underline{Y}_P)] + N_P.v(\underline{Y}_P)]$$

The first order conditions are:

$$v'(\overline{Y}_P) = \frac{1}{1-\overline{\theta}}, \qquad e$$

$$(1 - \underline{\gamma}).v'(\underline{Y}_P) \geq 1 + \frac{\gamma}{1 - \gamma}(\bar{\gamma} - \underline{\gamma}).v'(\underline{Y}_P)$$

Remember that in the case with complete information, we had:

$$v'(\bar{Y}_P^*) = \frac{1}{1 - \bar{\gamma}} \qquad e$$

$$(1 - \underline{\gamma}).v'(\underline{Y}_P^*) = 1$$

And thus, we can state that:

Proposition 5: with incomplete information, the poor under a government of a type more averse to poverty are as well off as they would be with complete information. However, the poor under a government less concerned with social issues are in a worse situation.

3.2.2 – Type Intervals

Let us consider the situation in which the municipality is of the type $\gamma \in [\underline{\gamma}, \bar{\gamma}]$. The municipality's type is private information, however, the function $f(\gamma)$ is of general knowledge.

The government would like to establish a contract with the municipality where the transfer value, T, depends on the accomplishment of certain pre-determined social targets, that is, a contract of the type $T=T(Y_P)$, assuming we are dealing with income targets, for example.

Such contract should establish differentiated targets according to the type of municipality. As this is unknown information to the government, it is up to the government to establish contracts $(Y_P, T(Y_P))$, and wait for the municipality's choice. This is equivalent to a revelation mechanism associating to each type announce by the municipality, a transfer $T(\hat{\gamma})$ for the income target $Y_P(\hat{\gamma})$.

The government's problem is to determine $T(\gamma)$ and $Y_P(\gamma)$, for each type γ, so as to maximize its utility, taking into consideration a distribution of types given by $f(\gamma)$.

$$\underset{Y_P(.),T(.)}{Max} \int_{\gamma^*}^{\bar{\gamma}} [G_F + N_P.v(Y_P(\gamma))]dF(\gamma)$$

$$s.a: \quad G_M(\gamma) + N_P.\gamma.v(Y_P(\gamma)) \geq U(\gamma) \qquad \gamma \in [\underline{\gamma}, \bar{\gamma}] \quad (RP\,\gamma)$$

$$G_M(\gamma) + N_P.\gamma.v(Y_P(\gamma)) \geq G_M(\hat{\gamma}) + N_P.\gamma.v(Y_P(\hat{\gamma})) \qquad \gamma, \hat{\gamma} \quad (RCI\,\gamma)$$

$$Y_F + T(\gamma) \geq N_P.v(Y_P(\gamma)) \geq Y_F + N_P.v(Y_P^A(\gamma)) \qquad (RP\ Governo)$$

The first restriction states that municipalities will only agree to a contract with the government if the utility derived from the contract is greater than or equal to the saved utility, which would be obtained if there had not been any contract, that is, in autarchy.

The second restriction guarantees the municipality the utility obtained when revealing that its true type $?$ is greater than that it would have obtained in case it had identified itself as being another type $\hat{?}$. This is the well-known Incentive Compatibility Restriction of type $?$.

The third and last restriction is so that the government can identify with which municipalities it is worth to establish a contract. It guarantees that the government's utility in carrying out the contract will be greater than if there had not been one. Nothing guarantees that it will be favorable for the principal (government) to establish a contract with all agents (municipalities), when there are an infinite number of types. In relation to the municipalities with low adversity to poverty, it maybe happen that it is not favorable for the government to perform transfers, for the municipality would invest a small amount in social programs, when compared to other municipalities more adverse to poverty. The type $?^*$ identifies the limit from which it is interesting for the government to transfer resources or not. This characteristic within the contract allows us to state that:

Proposition 6: the municipalities experiencing a more intense poverty—due to the low aversion to poverty by their local governments—can be impeded of signing contracts of social targets and thus be kept from receiving government funds.

This is a controversial result, since where the government is expected to intervene is actually the place where it should pass on the responsibility. As in the case of unconditional transfers, what happens is that in these municipalities the transfers performed by the government to the municipality almost does not change the poor's situation, since the municipality tends to reduce the channeling of its own resources to the social realm in a quantity almost equivalent to that received from the government[6].

Taking into consideration the definitions of G_F, G_M and $U(?)$, we can rewrite the equation of maximizing the government as:

[6] In practice, this problem is softened since part of the investments in the social area (education, health, social services, etc) has a minimum percentage linked to the local budget—refer to the Fiscal Responsibility Law and the Federal Constitution. This way, when the budget increases, the municipality is forced to increase its total expenses in these areas, incapable of simply using the federal funds and reducing the local ones by an equivalent amount.

$$\underset{Y_P(.),T(.)}{Max} \int_{?^*}^{\bar{?}} [Y_F - T(?) - N_P.v(Y_P(?))]dF(?)$$

s.a : $[Y_M - T(?) - N_P.Y_P(?)] - N_P.?.v(Y_P(?)) \geq [Y_M - N_P.Y_P^A(?)] - N_P.?.v(Y_P^A(?))$

$[Y_M - T(?) - N_P.Y_P(?)] - N_P.?.v(Y_P(?)) \geq [Y_M - T(\hat{?}) - N_P.Y_P(\hat{?})] - N_P.?.v(Y_P(\hat{?}))$

$Y_F - T(?) - N_P.v(Y_P(?)) \geq Y_F - N_P.v(Y_P^A(?))$

Defining the municipality of type $?$'s utility upon announcing itself as type $\hat{?}$ and choosing a contract $(Y_P(\hat{?}), T(\hat{?}))$, as $V(?,\hat{?})$, we have that:

$$V(?,\hat{?}) \equiv [Y_M - N_P.Y_P(\hat{?}) - T(\hat{?})] - N_P.?.v(Y_P(\hat{?}))$$

and defining $V(?)$ as the utility from revealing its true type :

$$V(?) \equiv V(?,?) \equiv [Y_M - N_P.Y_P(?) - T(?)] - N_P.?.v(Y_P(?))$$

This way, we can redefine the government's problem as:

$$\underset{Y_P(.),V(.)}{Max} \int_{?^*}^{\bar{?}} \{[Y_F - V(?) - Y_M - N_P.Y_P(?) - N_P.?.v(Y_P(?))] - N_P.v(Y_P(?))\}dF(?)$$

s.a : $V(?) \geq U(?)$ $\quad ? ? ? [\underline{?},\bar{?}]$ (RP $?$)

$V(?,?) \geq V(?,\hat{?})$ $\quad ? ? ? \hat{?}$ (RCI $?$)

$Y_F - [V(?) - Y_M - N_P.Y_P(?) - N_P.?.v(Y_P(?))] - N_P.v(Y_P(?)) \geq Y_F - N_P.v(Y_P^A(?))$

Solving the following equation, we have that:

Proposition 7: the optimum contract to be established between the government and a municipality of type $? ? ?^*$, given that $\frac{d}{dx}\left[\frac{1 - F(?)}{f(?)}\right] < 0$, may be characterized by:

a) $(1 - ?) - \frac{1 - F(?)}{f(?)}.v'(Y_P(?)) = 1$

b) $T(?) \equiv V(?) - Y_M - N_P.Y_P(?) - N_P.?.v(Y_P(?)) \quad ? ? ? [?^*,\bar{?}]$

where $V(?) \equiv \int_{?^*}^{\bar{?}} N_P.v(Y_P(?))d? \equiv U(?^*)$, and the coefficient $?^*$'s value is determined by the government's Participation Restriction

Proof: Appendix II

4 – Dynamic Model

One of the important aspects to be considered in contractual relations is the temporal dimension. Contracts are established and have deadlines for various periods—in general[7]. Up until now, we had analyzed only static contracts, observed only throughout one period. The objective in this chapter is to study the modifications occurring in our model when we deal with relations lasting over one period. We wish to know what type of contract should the government establish with the municipality having in mind long run actions, which could correspond to various term years, or even various terms.

We will support ourselves primarily based on the presentation regarding dynamic models made by Salanié (1997). We see that the results in the dynamic case are often contrary to what we would have expected in a more superficial analysis. In some cases, we limit ourselves to showing the result's subjacent intuition, without presenting a formal development, in light of the complexity of the dynamic models.

We restrict our analysis to **complete contracts**. These, according to Salanié, are those in which "all variables which may have an impact on the contractual relations' conditions, throughout its entirety, were taken into account at the moment of negotiation and the signing of the contract. This way, the contract should be contingent upon a large number of variables. This hypothesis implies that none unexpected situation appears during the contractual relation: any shift in the economic environment has as its only implication the implementation of a pre-established rule of the contract."

The hypothesis of complete contracts is relatively strong, although it displays the advantage of being reasonably studied. At the end of this chapter, we write a brief explanation for the implications of having incomplete contracts.

Commitment and **renegotiation** are two key concepts to our analysis. According to Salanié (1997), commitment refers to the agent's ability to restrict beforehand its future actions through the promise of maintaining the contract during an agreed period. The length of commitment determines the contract's strictness; the longer the length of the agent's commitment, the stricter the contract. An agent's commitment depends upon a series of factors, such as:

- ?? agent credibility: the greater the importance of an agent's reputation, the greater will be his commitment in keeping the contract, aiming to maintain or increase his reputation;

- ?? legal framework supporting the contracts: establishes punishments and fines in case of a contract breach;

[7] The definition of what is a period depends upon the situation; it can correspond to a month, a year, a mandate, a generation, etc.

?? contractual penalizations: should be applied, according to contract, in case it is breached unilaterally.

In counterpart to commitment, we have renegotiation and unilateral breach of contract. Renegotiation refers to the decision taken in overall agreement, bilateral or multilateral, of not fulfilling the contract terms previously agreed upon. The unilateral decision occurs when an agent does not keep the deal, without the attainment of any type of agreement from the other entities. Such a decision may lead to a fine, which does not occur in the previous case.

There are three distinct cases of what compromises the issue:

?? Full Commitment: the contract establishes the rules enduring throughout its lifetime, when there is not the possibility of any type of renegotiation among the parties, even if they agree about a change. Suppose, for example, that the contract involves three or more entities, and if two of them have the possibility of obtaining a mutual improvement in case there is a renegotiation. Even if this renegotiation does not worsen the situation of other entities, nonetheless it will not be allowed in a full commitment contract.

?? Long term commitment[8]: the contract establishes rules for all periods of its lifetime, however, having the possibility that the contract's members renegotiate their relations. Such a renegotiation is only possible if both parties are in agreement, not being permissible that one imposes upon the other a new contract. This type of contract is also known as long-term commitment with renegotiation.

?? No commitment or spot commitment: the contract establishes rules for the first period. In relation to the following periods, the parties may choose to sign a new contract with the same terms, different terms, or not sign a contract at all.

The issue of whether or not there exists commitment and the possibility for renegotiation among agents is fundamental in the analysis of complete dynamic contracts. Still referring to Salanié (1997), a final result in the theory of individual choices is that no agent, alone, can improve its situation by having its choice possibility limited. When there is a greater number of a choice restriction, the final result tends to be worsen—it might be the same, but never better. Such a result is not valid when there is interaction among agents. As an illustrative example, we have the Prisoner's Dilemma. The prisoners may declare themselves guilty or innocent and the resulting Nash equilibrium is that both declare themselves to be guilty. However, if both had committed to declare themselves as innocent, the result would be better for both. This shows that the existence of a commitment mechanism—implying a limitation in the prisoners' choice—would cause them to be better. The lack of commitment by the agents, however, becomes harmful to both. In relation to our model's dynamic contracts, we see the same principal being valid in the relationship between the government and municipalities.

[8] Dewatripont (1989) introduced the concept of long-term commitment.

4.1 – Full Commitment

Again suppose that the government is in a situation of incomplete information

Suppose that the government is in a situation of incomplete information, in which the type of the municipal administration with which it intends to establish a contract of social targets is unknown. The government knows there are two available types, $\overline{?}$ and $\underline{?}$, and the probabilities associated to each type are (1-?) and ?, respectively. This same problem was dealt with previously. Let's consider that the contract to be established between the government and the municipality has a due date of T periods instead of only one period (static case). Such contract cannot be renegotiated by any of the entities, be it unilaterally or bilaterally, even if such negotiation is consensual among them. In each period, the government takes on the commitment of performing a transfer T, for the municipality to invest in the social area, which is responsible for reaching a social target within each period.

The government's utility throughout the contract's lifetime is

$$U_F ? \sum_{t?1}^{T} ?^{t?1}(Y_{F_t} ? T_t) ? N_P.v(Y_{P_t})$$

and that of the local government is given by:

$$U_M ? \sum_{t?1}^{T} ?^{t?1}(Y_{M_t} ? T_t ? N_P.Y_{P_t}) ? N_P.?.v(Y_{P_t})$$

where ? is the inter-temporal discount factor, considered constant throughout time and the same for both government and municipality.

According to Salanié (1997), having total commitment, the revelation principle is valid in the dynamic case, for all parts interested in the contract negotiate once, when there are not types of alteration in the agreement.

This way, the government's problem is to propose, for each possible type of municipality, a sequence of targets and transfers for each contract year. It is the municipality's responsibility to announce itself as being $\overline{?}$ or $\underline{?}$ and sign the contract for its type. The government's problem, however, is to choose the sequence $\{Y_{P_t}(\overline{?}), T_t(\overline{?}), Y_{P_t}(\underline{?}), T_t(\underline{?})\}_{t?1}^{T}$ that maximizes its utility and that fulfills the restrictions of incentive compatibility and of municipal participation, such that it announces its true type.

In formal terms, the government's equation is given by:

$$\underset{(\bar{Y}_{P_t},\bar{T}_t,\underline{Y}_{P_t},\underline{T}_t)_{t?1}^{T}}{\text{Max}} \; ?.\left[\sum_{t?1}^{T} ?^{t?1}(Y_{F_t} ? \underline{T}_t) ? N_P.v(\underline{Y}_{P_t})\right] ? (1? ?).\left[\sum_{t?1}^{T} ?^{t?1}(Y_{F_t} ? \bar{T}_t) ? N_P.v(\bar{Y}_{P_t})\right]$$

s.a : (RP $\bar{?}$) $\sum_{t?1}^{T} ?^{t?1}.[(Y_{M_t} ? \bar{T}_t ? N_P.\bar{Y}_{P_t}) ? N_P.\bar{?}.v(\bar{Y}_{P_t})] ? U(\bar{?})$

(RP $\underline{?}$) $\sum_{t?1}^{T} ?^{t?1}.[(Y_{M_t} ? \underline{T}_t ? N_P.\underline{Y}_{P_t}) ? N_P.\underline{?}.v(\underline{Y}_{P_t})] ? U(\underline{?})$

(RCI $\bar{?}$) $\sum_{t?1}^{T} ?^{t?1}.[(Y_{M_t} ? \bar{T}_t ? N_P.\bar{Y}_{P_t}) ? N_P.\bar{?}.v(\bar{Y}_{P_t})] ? \sum_{t?1}^{T} ?^{t?1}.[(Y_{M_t} ? \underline{T}_t ? N_P.\underline{Y}_{P_t}) ? N_P.\bar{?}.v(\underline{Y}_{P_t})]$

(RCI $\underline{?}$) $\sum_{t?1}^{T} ?^{t?1}.[(Y_{M_t} ? \underline{T}_t ? N_P.\underline{Y}_{P_t}) ? N_P.\underline{?}.v(\underline{Y}_{P_t})] ? \sum_{t?1}^{T} ?^{t?1}.[(Y_{M_t} ? \bar{T}_t ? N_P.\bar{Y}_{P_t}) ? N_P.\underline{?}.v(\bar{Y}_{P_t})]$

The solution to this equation allows us to establish that:

Proposition 8: having total commitment, the government must establish as target to be reached by the municipality, the same that would be established in the static case (one period). This target must be maintained throughout the contract's lifetime—during T periods. The optimum contract has the following sequence of targets and transfers:

$$\{Y_{P_t}(\bar{?}), T_t(\bar{?}), Y_{P_t}(\underline{?}), T_t(\underline{?})\}_{t?1}^{T} ? \{\bar{Y}_P, \bar{T}, \underline{Y}_P, \underline{T}\}_{t?1}^{T}$$

where $\{\bar{Y}_P, \bar{T}, \underline{Y}_P, \underline{T}\}$ is the solution to the static case.

Proof: Appendix III

The process occurs as if an optimum contract for a single period had been established and this contract had been continuously renewed during T periods. Some possible interpretations for this result are:

a) If the target Y_P is an income target, the government's objective should be to establish minimum income, $\bar{Y}_P$ e $\underline{Y}_P$ —which should be reached by the first year— for each type of municipality, transferring $\bar{T}$ e $\underline{T}$ each year, as a way of maintaining the minimum income.
b) If the target Y_P is seen as a percentage variation—for example, the reduction of infant mortality rate, the increase in school attendance—the government's objective becomes the attainment of a continuous variation over the chosen social indicator, such that period after period, it is the same as the one obtained in the first period.

Figure 1 below displays the solution to the problem when we have a contract expanding over only two periods.

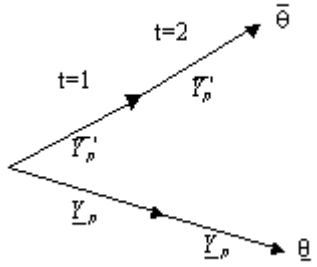

Figure 1

The problem of total commitment contracts is how to guarantee that there are not bilateral negotiations taking place. In our case, after the initial period, the municipalities reveal their types and the government starts to have an incentive to propose a renegotiation with some municipalities. We cannot forget that, due to the asymmetry of information, the contract established between the government and the municipality of type $\underline{?}$, $(\underline{Y}_P, \underline{T})$, is done so in a way that the municipality has as its target a value lower than that established by complete information $\underline{Y}_P$? $\underline{Y}_P^*$. This causes an inefficient allocation of public resources. Since part of the informational asymmetry disappears after the first period, the government would like to propose to the municipality of type $\underline{?}$, in the second period, an optimum contract ($\underline{Y}_P^*, \underline{T}^*$).In this type of contract, the municipality has a higher target to accomplish and would receive more resources to do so, such that its utility would remains the same. However, the government would be better off and so would the poor. This type of reasoning suggests that the establishment of a contract with total commitment consequently has ex-post inefficiency, in light that the entities are kept from renegotiating among themselves. What would occur if this possibility were allowed? That is what we will determine in the following item.

4.2 – Long Term Commitment

Let's suppose that the only difference to the previous case is that we deal with a two-period dynamic contract, instead of T periods[9]. Besides, we have the possibility of a bilateral or multilateral renegotiation, if there is a consensus among the parties, since the contract is a long-term commitment one.

In this situation, the government knows the type of each municipality after the first period, in accordance to the chosen contract. However, there is a problem of complete information for the second period, in which the government would like to establish new contracts with all municipalities, using the information it has acquired about each one. It would be ideal for the government to establish an optimum contract (first-best) in the second period. However, with this type of contract, the municipality of type $\overline{?}$ would experience a loss in utility. As has been stated, one of the conditions so that renegotiation occurs is that both

[9] Due to the complexity of the dynamic problem, we use the usual approach, which consists in analyzing the problem with two periods.

parties are in agreement. Obviously the municipality of type $\overline{?}$ would not agree to renegotiate its contract if that meant establishing an optimum contract for the government.

In relation to the municipality of type $\underline{?}$, if the government offered an optimum contract, the municipality would not be better nor worse—remember that in both the optimum contract and in the one with incomplete information, the municipality of type $\underline{?}$ has the same utility (reserved utility) as obtained in autarchy. This way, the municipality would be willing to accept the new contract, resulting in amelioration for the government and the poor. In this situation, however, there would be incentive so that a renegotiation would occur between the government and the municipality of type $\underline{?}$.

At first sight, the contract with long-term commitment allows for a gain in efficiency in the use of public money. Such a conclusion, however, is not so simple. Let's understand why.

As observed in the problem with two types of municipalities and incomplete information, the municipality of type $\overline{?}$ has a likelihood of pretending to be of type $\underline{?}$, So that this does not occur, the government maximizes its utility subject to incentive compatibility restrictions, and proposes a menu of contracts so that the municipalities reveal their true type. The solution to the problem implies that the municipality $\overline{?}$ obtains an informational income and is indifferent between a contract of its type and of type $\underline{?}$—we suppose that when the municipality is indifferent, it chooses the contract for its type. Another characteristic of this menu is that municipality $\underline{?}$ obtains a contract in which it must reach a target below the optimum target, for if a contract were offered in which municipality $\underline{?}$ had to reach the optimum target, then the municipality $\overline{?}$ would pretend to be of type $\underline{?}$.

In the dynamic case, we see that it is advantageous for the government to renegotiate with municipality $\underline{?}$ in the second period, and offer an optimum contract. What happens is that municipality of type $\overline{?}$, knowing that there is such a possibility in the second period, prefers to pretend to be of type $\underline{?}$ in the first period. The reason for this is that:

- In relation to the first period, its utility will not change.
- In the second period, however, its utility will increase. In the beginning of the second period, the government will think that it's of type $\underline{?}$ and will propose a contract renegotiation, offering an optimum contract for type $\underline{?}$. This contract, as explained, provides a greater utility than that obtained with the contract offered to type $\overline{?}$ in the first period.

The result is that the government, by establishing a contract allowing renegotiation, motivates the municipalities of type $\overline{?}$ to not reveal its type and to make themselves pretend to be less concerned with poverty, $\underline{?}$. This creates a hardship in the choice of municipalities $\overline{?}$ of contracts having modest social targets than those they would have chosen had they known there would not be a renegotiation between the government and municipalities of type $\underline{?}$. Therefore, what appears to be a solution to increase the efficiency of public funds, ends up being a source of greater inefficiency.

A contract with full commitment is inefficient ex-post to the government when comparing with the long-term commitment contract, since the government does not use the information obtained from the first period in the second period. However, the long-term commitment contract is inefficient ex-ante in relation to the full commitment one, for as long as there is no commitment, the final result is worse for the government.

What the theory shows us is that to find a solution to the contract with long-term commitment it is necessary to consider, in the formulation of the problem, the possibility of renegotiation. This is done through the inclusion of additional restrictions, known as sequential efficiency restrictions or non-renegotiation restrictions. This denomination occurs due to the fact that solutions obtained with these restrictions imply that there is no renegotiation during the contract's lifetime. Any possible renegotiation is anticipated and considered at the moment of the contract's elaboration.

Solutions of this type are extremely complex. Due to this, we base ourselves on articles dealing with similar problems to derive the type of solution we might find in our model. Hart-Tirole (1988) and Laffont-Tirole (1990)—considering a contract with two periods—solve the problem of dynamic long-term commitment contracts in different contexts. In the solutions found, in the first period, agents of type $\overline{?}$ split. One part, 1-x, reveals its type, while the other, x, pretends to be $\underline{?}$. For those revealing their type, the principal offers an optimum contract with incomplete information $(\overline{Y}_P,\overline{T})$. In the second period, the agents of type $\overline{?}$ pretending to be $\underline{?}$, reveal their type, renegotiate the contract and sign the same type of contract $(\overline{Y}_P,\overline{T})$ as other agents of type $\overline{?}$ had already signed in the first period.

Following, in figure 2, we illustrate the type of solution found in the cited articles. In our case, considering the probability that the municipality is of type $\overline{?}$ is $?$ and that the portion of municipalities not revealing their type is x, then at the beginning of the second period, the probability of a municipality being of type $\overline{?}$ (in case it had identified itself as $\underline{?}$ in the first period) is:

$$?_2 \; ? \; \frac{? .x}{? .x ? (1 ? ?)}$$

Considering that the second period is also the last, the solution in this period is determined as the solution to the static problem. This way, the contract offered to the type $\underline{?}$ in the second period is equal to the solution to the problem with two types of municipalities and incomplete information. The only thing needed is to substitute the probability $?$ for the probability $?_2$ in the first order condition determined by that case. The FOC attained in the second period is:

$$(1 ? \underline{?}).v'(\underline{Y}_P) \; ? \; 1 ? \frac{?_2}{1 ? ?_2}\left(\overline{?} ? \underline{?}\right).v'(\underline{Y}_P)$$

Given that $?_2 \; ? \; ?$ we have that:

$\underline{Y}_{P2}$? $\underline{Y}_{P}$

We see, however, that the possibility of the government to renegotiate—with a municipality of type ?—the contract in the second period, implies in a solution with higher targets for these municipalities. Having in mind the targets as the poor's income, there is an increase in the poorest's income. This does not mean an increase in the efficiency of the public money use, since part of municipalities ? pretends to be of type ? and reaches lower targets in the first period that in case of full commitment. Besides, the targets of type ? in the first period are lower than they would have been with full commitment.

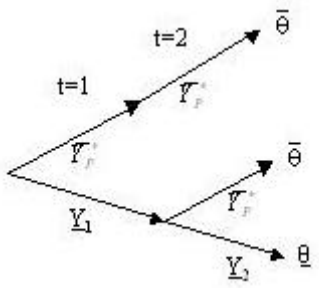

Figure 2

4.3 – Non-commitment

In this case, the government does not have the commitment to maintain in the second period the contract established in the first. In a long-term commitment contract, if the municipality of type ? revealed its type in the first period, it would have ensured in the second period the same contract as in the previous period. This would then guarantee an informational income equal to that in the first period, since the government would be unable to use the information obtained to impose a renegotiation implying losses for the municipality.

In the case of non-commitment, the government, once aware of the type of municipality, is not obliged to repeat in the second period the initial contract. More than this, it can use the information attained in the first period and offer an optimum contract (first-best) as the only alternative to the municipality of type ?. This implies that the municipality of this type attains an informational income equal to zero in the second period and a utility equal to that obtained in autarchy. Due to this possibility, the municipality of type ? prefers to identify itself as being ? in the first period. In this case, its utility in the first period does not change—having the same informational income it would have had it identified itself as ?--and it may acquire an informational income also in the second period, having seen that the government remains not knowing its type. As a result, in this type of contract, inefficiency is even greater than the long-term commitment, since the incentive for the municipality of type ? to choose the contract of type ? are even greater than in the previous case.

In the case in which the government has the freedom of making total use of all the information attained in the first period, the result is the worst possible, since the municipality of type $\overline{\theta}$ does everything in its power so as to not reveal any information, or reveal information in the slowest manner possible. This is the known ratchet effect, for once the municipality reveals any information regarding its type, it permanently loses the possibility of having some sort of informational income with this information, being unable to turn back time.

To avoid that the municipality $\overline{\theta}$ identifies itself as $\underline{\theta}$, the government must anticipate—in the first period—all expected value for informational income that $\overline{\theta}$ might obtain in the future had there been a commitment, discounting according to the parameter d. The problem with this type of solution is that the help given in the first period to those identifying themselves $\overline{\theta}$ as can be so high that it induces the municipality of type $\underline{\theta}$ to pretend to be $\overline{\theta}$. So that this does not occur, the government must find a gray area, so that in a contract with T periods, the municipality slowly reveals its type.

Problems of this type are extremely difficult to solve. We will restrain ourselves only in the explanation for the intuition. As Salanié (1997) summarizes, the velocity of revealing the type depends primarily on the parameters d e T. In a situation where the mandate is almost over—when the mayor is not concerned with the future or has a low commitment to the future administration—we have a situation with a low d or even equal to zero. In this case, the velocity of revealing information is high. In the opposite case—the beginning of a term—an agreed contract with the possibility of being renewed throughout the term, causes the municipality to slowly reveal its type[10].

The situation without any type of commitment displays slower velocity of type revelation, which implies a greater inefficiency in the allocation of public resources.

Summarizing the issue of dynamic problem, we have that:

Proposition 9: In a situation with full contracts and incomplete information, the best the government can do to increase the efficiency of public funds is to offer an optimum contract with incomplete information throughout the course of contract's duration, creating institutional mechanisms guaranteeing the impossibility of bilateral negotiations.

4.4 – Incomplete Contracts

In the prior section, we concluded that under the hypothesis of **full contract**, the ideal is that the government establish a pact with all participating municipalities, so that during the social target contract's lifetime, there is not the possibility of the government bilaterally renegotiating the targets with some municipalities. Such as would occur in the Prisoner's Dilemma, the alternatives' restriction imposed by full commitment allows for a Pareto improvement in relation to other solutions.

[10] Laffont-Tirole (1987) analyzed the comparative statistic of optimum contracts in the case of dynamic incentives.

However, this conclusion remains invalid in the case where we have incomplete contracts. This is an important implication, since the hypothesis of full contracts is relatively strong. In the real world, there are a series of problems to attain a full contract:

?? The elaboration of a contract has expenses. In some situations, the cost of contemplating an unlikely situation can be greater than the benefit of farseeing what to do in that situation;
?? In some contingent state, the verification of the values taken on by relevant variables is difficult or impossible. It does not allow for a mediation of disputes potentially arising;
?? There is a problem of limited rationality which makes the agents unaware of the proper knowledge to precisely evaluate the impact of some variables;
?? There is a difficulty and even impossibility in attributing probabilities for every natural state.

In the previous case, the possibility of renegotiation created ex-ante inefficiencies. Meanwhile, in this situation, the renegotiation functions as a means of treating the cases unpredicted in the contract, which could bring social gains.

5 – Conclusion

[write conclusion]

6 –References

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Appendix I – Proposition 3

So that the poor experience an increase of one unit in income, it is necessary that the municipality spend $N_P.1$ in the social project. According to the targets' contract, for each unit of increase in the poor's income, the government transfers to the municipality $b.N_P$ in funds. The liquid result is a variation on the local available budget for increment unit in the poor's income:

$$\frac{?G_M}{?Y_P} ? b.N_P ? N_p \qquad (*)$$

The government's utility function:

$$U_F ? Y_F ? T^C(Y_P) ? N_P.v(Y_P)$$

From the municipality's budgetary restriction (RO), we have:

$$Y_M ? T^C(Y_P) ? G_M ? N_P.Y_P$$

Isolating $T^C(Y_P)$ in RO and substituting the government's utility function, we have that:

$$G_M ? (Y_F ? U_F ? Y_M) ? N_P.Y_P ? N_P.v(Y_P)$$

In optimum, we have:

$$\frac{dG_M}{dY_P} ? ?N_P ? N_P.v'(Y_P^{TC}) \qquad \text{onde,} \quad v'(Y_P^{TC}) ? \frac{1}{1??}$$

Thus,

$$\frac{dG_M}{dY_P} ? ?N_P ? N_P.\frac{1}{1??} \qquad (**)$$

From (*) and (**) we have:

$$b.N_P ? N_p ? ?N_P ? N_P.\frac{1}{1??} \quad ? \quad b ? \frac{1}{1??}$$

In relation to the coefficient "a", we have:

$T(Y_P^{TC}) ? a ? b.Y_P^{TC}$

Logo,

$a ? T(Y_P^{TC}) ? b.Y_P^{TC}$

Being that

$b ? \frac{1}{1 ? ?}, \quad v'(Y_P^{TC}) ? \frac{1}{1 ? ?}, \quad T(Y_P^{TC}) ? N_P.[(Y_P^{TC} ? Y_P^A) ? ?.(v(Y_P^{TC}) ? v(Y_P^A))]$

Appendix II – Proposition 7

The government's problem is:

$$\underset{Y_P(.),V(.)}{Max} \int_{?^*}^{\bar{?}} \{[Y_F ? V(?) ? Y_M ? ? N_P.Y_P(?) ? N_P.?.v(Y_P(?))] ? N_P.v(Y_P(?))\} dF(?)$$

$$s.a: \quad V(?) ? U(?) \qquad ? ? ? [\underline{?}, \bar{?}] \quad (RP?)$$

$$V(?,?) ? V(?,\hat{?}) \qquad ? ? ? \hat{?} \qquad (RCI?)$$

$$Y_F ? [V(?) ? Y_M ? N_P.Y_P(?) ? N_P.?.v(Y_P(?))] ? N_P.v(Y_P(?)) ? Y_F ? N_P.v(Y_P^A(?))$$

Analyzing the incentive compatibility restriction, we see that the for the local utility to be maximum when revealing its true type, it is necessary that:

$$\left.\frac{?V(?,\hat{?})}{??}\right|_{\hat{???}} ? 0 \qquad e \qquad \left.\frac{?^2V(?,\hat{?})}{??^2}\right|_{\hat{???}} ? 0$$

Considering that: $V(?,\hat{?}) ? [Y_M ? N_P.Y_P(\hat{?}) ? T(\hat{?})] ? N_P.?.v(Y_P(\hat{?}))$

$$\frac{?V(?,\hat{?})}{??} ? ? N_P.Y_P'(\hat{?}) ? T'(\hat{?}) ? N_P.?.v'(Y_P(\hat{?})).Y_P'(\hat{?}) \qquad (1)$$

$$\frac{?^2V(?,\hat{?})}{??^2} ? ? N_P.Y_P''(\hat{?}) ? T''(\hat{?}) ? N_P.?.[v''(Y_P(\hat{?})).Y_P'(\hat{?}).Y_P'(\hat{?}) ? v'(Y_P(\hat{?})).Y_P''(\hat{?})] \qquad (2)$$

Therefore,

$$\left.\frac{?V(?,\hat{?})}{??}\right|_{\hat{???}} ? 0 \quad ? \quad T'(?) ? N_P.Y_P'(?) ? N_P.?.v'(Y_P(?)).Y_P'(?) \qquad (3)$$

$$\left.\frac{?^2V(?,\hat{?})}{??\hat{?}^2}\right|_{\hat{?}??} ? 0 \; ? \quad T''(?) ? N_P.Y_P''(?) ? N_P.?.[v''(Y_P(?)).(Y_P'(?))^2 ? v'(Y_P(?)).Y_P''(?)]$$

(4)

Deriving (3) in relation ? we have:

$$T''(?) ? N_P.Y_P''(?) ? N_P.?.[v''(Y_P(?)).(Y_P'(?))^2 ? v'(Y_P(?)).Y_P''(?)] ? N_P v'(Y_P(?)).Y_P'(?) \quad (5)$$

lado direito da equação (4)

Substituting (5) in (4):

$$T''(?) ? T''(?) ? N_P.v'(Y_P(?)).Y_P'(?) \; ? \; v'(Y_P(?)).Y_P'(?) ? 0$$

Given that $v'(Y_P(?)) ? 0$ and thus

$$\boxed{Y_P'(?) ? 0} \qquad (4')$$

It was defined that: $V(?) ? [Y_M ? N_P.Y_P(?) ? T(?)] ? N_P.?.v(Y_P(?))$. Deriving this equation in relation to ? we have:

$$V'(?) ? ?N_P.Y_P'(?) ? T'(?) ? N_P.v(Y_P(?)) ? N_P.?.v'(Y_P(?)).Y_P(?) ?$$
$$T'(?) ? V'(?) ? N_P.v(Y_P(?)) ? N_P.Y_P'(?) ? N_P.?.v'(Y_P(?)).Y_P(?) \qquad (6)$$

lado direito da expressão (3)

Substituting (6) in (3):

$$\boxed{V'(?) ? N_P.v(Y_P(?))} \qquad (3')$$

However, the incentive compatibility restriction of a municipality of type ? (RCI ?) can be substituted by the equations (3´) and (4´) in the government's problem.

The government's problem with the new restrictions is:

$$\underset{Y_P(.),V(.)}{Max} \int_{?^*}^{\bar{?}} \{[Y_F ? V(?) ? Y_M ? ?N_P.Y_P(?) ? N_P.?.v(Y_P(?))] ? N_P.v(Y_P(?))\} dF(?)$$

s.a : $V(?) ? U(?) \qquad ??? [\underline{?}, \bar{?}]$ (RP ?)

$Y_P'(?) ? 0$

$V'(?) ? N_P.v(Y_P(?))$

$Y_F ? [V(?) ? Y_M ? N_P.Y_P(?) ? N_P.?.v(Y_P(?))] ? N_P.v(Y_P(?)) ? Y_F ? N_P.v(Y_P^A(?))$

The equation's Hamiltonian is given by:

$$H = [Y_F - V(\tau) + Y_M - \tau N_p.Y_p(\tau) - N_p.\tau.v(Y_p(\tau))] - N_p.v(Y_p(\tau))].f(\tau) - \tau(\tau).N_p.v(Y_p(\tau))$$

$$\frac{\partial H}{\partial V} = \tau\tau'(\tau) \quad = \quad f(\tau) - \tau'(\tau) \quad = \quad \int_{\tau}^{\bar{\tau}} f(u).du - \int_{\tau}^{\bar{\tau}} f(u).du \quad = \quad \tau(\bar{\tau}) - \tau(\tau) - F(\bar{\tau}) + F(\tau)$$

Considering that for $\bar{\tau}$ the restriction is inactive, $\tau(\bar{\tau}) = 0$, and thus

$$\tau(\tau) = \tau(1 - F(\tau)) \qquad (9)$$

$$\frac{\partial H}{\partial Y_p} = 0 \quad = \quad [\tau N_p - N_p.\tau.v'(Y_p(\tau))] - N_p.v'(Y_p(\tau))].f(\tau) - \tau(\tau).N_p.v'(Y_p(\tau)) = 0 \quad =$$

$$v'(Y_p(\tau)).[(1 - \tau).f(\tau) - \tau(\tau)] - f(\tau) \qquad (10)$$

Substituting (9) in (10):

$$\boxed{v'(Y_p(\tau)).\left(\tau(1 - \tau) - \frac{1 - F(\tau)}{f(\tau)}\right) = 1}$$

b) The equation for the value to be transferred from the government to the municipality is obtained from the definition of $V(\tau)$:

$$V(\tau) = [Y_M - N_p.Y_p(\tau) - T(\tau)] - N_p.\tau.v(Y_p(\tau)) \quad =$$

$$\boxed{T(\tau) = V(\tau) - Y_M - N_p.Y_p(\tau) - N_p.\tau.v(Y_p(\tau))}$$

To obtain $V(\tau)$ take the integral of (3´):

$$\int_{\tau^*}^{\tau} V'(u).du - \int_{\tau^*}^{\tau} N_p.v(Y_p(u)).du \quad = \quad V(\tau) - V(\tau^*) - \int_{\tau^*}^{\tau} N_p.v(Y_p(u)).du \quad = \quad |V(\tau^*) - U(\tau^*)| \quad =$$

$$V(\tau) = \int_{\tau}^{\tau^*} N_p.v(Y_p(\tau)).du + U(\tau^*)$$

Appendix III – Proposition 8

$M ? \{Y_{P_t}(\overline{?}), T_t(\overline{?}), Y_{P_t}(\underline{?}), T_t(\underline{?})\}_{t?1}^{T}$ is the solution to the government's equation in the dynamic situation. Such optimum mechanisms should fulfill the incentive compatibility restrictions and the municipality's participation, that is:

$$\sum_{t?1}^{T} ?^{t?1}.[Y_{M_t} ? N_P.\overline{Y}_P ? \overline{T}_t ? N_p.\overline{?}.v(\overline{Y}_{P_t})] ? \sum_{t?1}^{T} ?^{t?1}.U(\overline{?}) \qquad \text{(RP } \overline{?}\text{)}$$

$$\sum_{t?1}^{T} ?^{t?1}.[Y_{M_t} ? N_P.\underline{Y}_P ? \underline{T}_t ? N_p.\underline{?}.v(\underline{Y}_{P_t})] ? \sum_{t?1}^{T} ?^{t?1}.U(\underline{?}) \qquad \text{(RP } \underline{?}\text{)}$$

$$\sum_{t?1}^{T} ?^{t?1}.[Y_{M_t} ? N_P.\overline{Y}_P ? \overline{T}_t ? N_p.\overline{?}.v(\overline{Y}_{P_t})] ? \sum_{t?1}^{T} ?^{t?1}.[Y_{M_t} ? N_P.\underline{Y}_P ? \underline{T}_t ? N_p.\overline{?}.v(\underline{Y}_{P_t})] \qquad \text{(RCI } \overline{?}\text{)}$$

$$\sum_{t?1}^{T} ?^{t?1}.[Y_{M_t} ? N_P.\underline{Y}_P ? \underline{T}_t ? N_p.\underline{?}.v(\underline{Y}_{P_t})] ? \sum_{t?1}^{T} ?^{t?1}.[Y_{M_t} ? N_P.\overline{Y}_P ? \overline{T}_t ? N_p.\underline{?}.v(\overline{Y}_{P_t})] \qquad \text{(RCI } \underline{?}\text{)}$$

We now consider the static model, which offers to the municipality a lottery of contracts, such that:

$(\overline{Y}_{P_1}, \overline{T}_1, \underline{Y}_{P_1}, \underline{T}_1)$ ocorra com probabilidade $\dfrac{1}{1 ? ? ? \ldots ? ?^{T?1}}$

$(\overline{Y}_{P_2}, T_2, \underline{Y}_{P_2}, \underline{T}_2)$ ocorra com probabilidade $\dfrac{?}{1 ? ? ? \ldots ? ?^{T?1}}$

.....

$(\overline{Y}_{P_T}, \overline{T}_T, \underline{Y}_{P_T}, \underline{T}_T)$ ocorra com probabilidade $\dfrac{?^{T?1}}{1 ? ? ? \ldots ? ?^{T?1}}$

If a municipality of type $\underline{?}$ accepts the contract lottery and reveals its true type, its expected utility is of:

$$\frac{1}{1 ? ? ? \ldots ? ?^{T?1}} ? Y_{M_1} ? N_P.\underline{Y}_{P_1} ? \underline{T}_1 ? N_P.\underline{?}.v(\underline{Y}_{P_1}) ? ? \ldots ? \frac{?^{T?1}}{1 ? ? ? \ldots ? ?^{T?1}} ? Y_{M_1} ? N_P.\underline{Y}_{P_1} ? \underline{T}_1 ? N_P.\underline{?}.v(\underline{Y}_{P_1}) ?$$

$$? \frac{1}{1 ? ? ? \ldots ? ?^{T?1}} ? \sum_{t?1}^{T} ?^{t?1}.? Y_{M_t} ? N_P.\underline{Y}_{P_t} ? \underline{T}_t ? N_P.\underline{?}.v(\underline{Y}_{P_t}) ? ?$$

$$\underset{RP\ \underline{?}\ \text{dinâmica}}{?} \frac{1}{1 ? ? ? \ldots ? ?^{T?1}} ? \sum_{t?1}^{T} ?^{t?1} U(\underline{?}) ? ?$$

$? U(\underline{?})$

This way, it is verified that the lottery fulfills the participation restriction of a municipality of type $\underline{?}$ in the static model. The verification for type $\overline{?}$ is analogous.

In relation to the incentive compatibility restriction for a municipality of type $\underline{?}$, we have that:

$$\frac{1}{1+r+r^2+\ldots+r^{T+1}}\left[Y_{M_1} + N_P.\underline{Y}_{P_1} + \underline{T}_1 + N_P.\underline{\tau}.v(\underline{Y}_{P_1})\right] + \ldots + \frac{r^{T+1}}{1+r+r^2+\ldots+r^{T+1}}\left[Y_{M_1} + N_P.\underline{Y}_{P_1} + \underline{T}_1 + N_P.\underline{\tau}.v(\underline{Y}_{P_1})\right]$$

$$+ \frac{1}{1+r+r^2+\ldots+r^{T+1}}\left[\sum_{t=1}^{T} r^{t+1}.\left[Y_{M_t} + N_P.\underline{Y}_{P_t} + \underline{T}_t + N_P.\underline{\tau}.v(\underline{Y}_{P_t})\right]\right]$$

$$\underset{\text{RCI}\,\underline{+}\,\text{dinâmica}}{+} \frac{1}{1+r+r^2+\ldots+r^{T+1}}\left[\sum_{t=1}^{T} r^{t+1}.\left[Y_{M_t} + N_P.\bar{Y}_P + \bar{T}_t + N_P.\underline{\tau}.v(\bar{Y}_{P_t})\right]\right]$$

For the government, the expected utility from the lottery is:

$$U_F + \tau.\left[\sum_{t=1}^{T} \frac{r^{t+1}}{1+r+r^2+\ldots+r^{T+1}}\left(Y_{F_t} + \underline{T}_t\right) + N_P.v(\underline{Y}_{P_t})\right] + (1+\tau).\left[\sum_{t=1}^{T} \frac{r^{t+1}}{1+r+r^2+\ldots+r^{T+1}}\left(Y_{F_t} + \bar{T}_t\right) + N_P.v(\bar{Y}_{P_t})\right]$$

$$+ \frac{1}{1+r+r^2+\ldots+r^{T+1}}\left[\tau.\sum_{t=1}^{T} r^{t+1}.\left(Y_{F_t} + \underline{T}_t\right) + N_P.v(\underline{Y}_{P_t})\right] + (1+\tau).\sum_{t=1}^{T} r^{t+1}.\left(Y_{F_t} + \bar{T}_t\right) + N_P.v(\bar{Y}_{P_t})$$

$$+ \tau.((Y_F + \underline{T}) + N_P.v(\underline{Y}_P)) + (1+\tau).((Y_F + \bar{T}) + N_P.v(\bar{Y}_P))$$

solução ótima do governo no caso estático com informação incompleta

Therefore,

$$\tau.\sum_{t=1}^{T} r^{t+1}.\left(Y_{F_t} + \underline{T}_t\right) + N_P.v(\underline{Y}_{P_t})\Big] + (1+\tau).\sum_{t=1}^{T} r^{t+1}.\left(Y_{F_t} + \bar{T}_t\right) + N_P.v(\bar{Y}_{P_t})\Big]$$

$$+ \tau.\sum_{t=1}^{T} r^{t+1}((Y_F + \underline{T}) + N_P.v(\underline{Y}_P)) + (1+\tau).\sum_{t=1}^{T} r^{t+1}((Y_F + \bar{T}) + N_P.v(\bar{Y}_P))$$

This way, the government's expected utility in the dynamic case cannot be greater than in the static case, being the same if the government repeats the static solution for each T periods of the contract's lifetime.