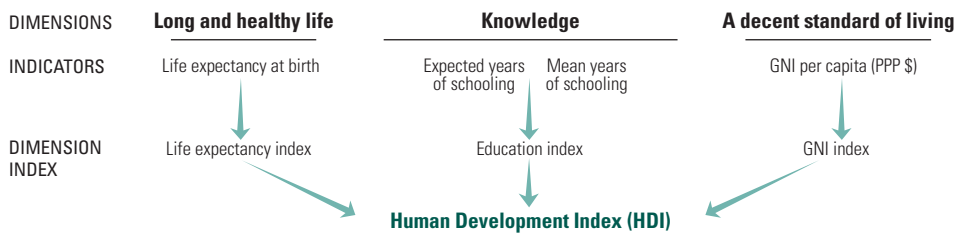
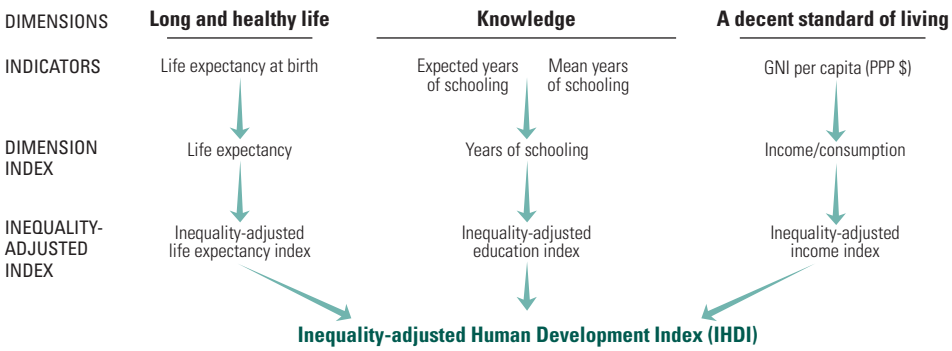
Technical notes

Calculating the human development indices—graphical presentation

Human Development Index (HDI)



Inequality-adjusted Human Development Index (IHDI)



Technical note 1. Human Development Index

The Human Development Index (HDI) is a summary measure of achievements in three key dimensions of human development: a long and healthy life, access to knowledge and a decent standard of living. The HDI is the geometric mean of normalized indices for each of the three dimensions.

Data sources

- Life expectancy at birth: UNDESA (2015).
- Expected years of schooling: UNESCO Institute for Statistics (2016), United Nations Children's Fund (UNICEF) Multiple Indicator Cluster Surveys and ICF Macro Demographic and Health Surveys.
- Mean years of schooling: Barro and Lee (2016), UNESCO Institute for Statistics (2016), Human Development Report Office updates based on UNESCO Institute for Statistics (2016), UNICEF Multiple Indicator Cluster Surveys and ICF Macro Demographic and Health Surveys.
- GNI per capita: IMF (2016), UNSD (2016) and World Bank (2016).

Steps to calculate the Human Development Index

There are two steps to calculating the HDI.

Step 1. Creating the dimension indices

Minimum and maximum values (goalposts) are set in order to transform the indicators expressed in different units into indices on a scale of 0 to 1. These goalposts act as the "natural zeros" and "aspirational targets," respectively, from which component indicators are standardized (see equation 1 below). They are set at the following values:

Dimension	Indicator	Minimum	Maximum
Health	Life expectancy (years)	20	85
Education	Expected years of schooling (years)	0	18
	Mean years of schooling (years)	0	15
Standard of living	Gross national income per capita (2011 PPP $)	100	75,000

The justification for placing the natural zero for life expectancy at 20 years is based on historical evidence that no country in the 20th century had a life expectancy of less than 20 years (Maddison 2010; Oeppen and Vaupel 2002; Riley 2005).

Societies can subsist without formal education, justifying the education minimum of 0 years. The maximum for expected years of schooling, 18, is equivalent to achieving a master's degree in most countries. The maximum for mean years of schooling, 15, is the projected maximum of this indicator for 2025.

The low minimum value for gross national income (GNI) per capita, $100, is justified by the considerable amount of unmeasured subsistence and nonmarket production in economies close to the minimum, which is not captured in the official data. The maximum is set at $75,000 per capita. Kahneman and Deaton (2010) have shown that there is virtually no gain in human development and well-being from income per capita above $75,000. Currently, only four countries (Kuwait, Liechtenstein, Qatar and Singapore) exceed the $75,000 income per capita ceiling.

Having defined the minimum and maximum values, the dimension indices are calculated as:

$$\text{Dimension index} = \frac{\text{actual value} - \text{minimum value}}{\text{maximum value} - \text{minimum value}}. \quad (1)$$

For the education dimension, equation 1 is first applied to each of the two indicators, and then the arithmetic mean of the two resulting indices is taken.

Because each dimension index is a proxy for capabilities in the corresponding dimension, the transformation function from income to capabilities is likely to be concave (Anand and Sen 2000)—that is, each additional dollar of income has a smaller effect on expanding capabilities. Thus for income the natural logarithm of the actual, minimum and maximum values is used.

Step 2. Aggregating the dimensional indices to produce the Human Development Index

The HDI is the geometric mean of the three dimension indices:

$$HDI = (I_{Health} \cdot I_{Education} \cdot I_{Income})^{1/3}$$

Example: Georgia

Indicator	Value
Life expectancy at birth (years)	75.0
Expected years of schooling (years)	13.9
Mean years of schooling (years)	12.2
Gross national income per capita (2011 PPP $)	8,856

Note: Values are rounded.

$$\text{Health index} = \frac{75.020 - 20}{85 - 20} = 0.8465$$

$$\text{Expected years of schooling index} = \frac{13.905 - 0}{18 - 0} = 0.77249$$

$$\text{Mean years of schooling index} = \frac{12.246 - 0}{15 - 0} = 0.81643$$

$$\text{Education index} = \frac{0.81643 + 0.77249}{2} = 0.7945$$

$$\text{Income index} = \frac{\ln(8{,}855.8) - \ln(100)}{\ln(75{,}000) - \ln(100)} = 0.6773$$

$$\text{Human Development Index} = (0.8465 \cdot 0.7945 \cdot 0.6773)^{1/3} = 0.769$$

Methodology used to express income

The World Bank's 2016 World Development Indicators database contains estimates of GNI per capita in constant 2011 purchasing power parity (PPP) terms for many countries. For countries missing this indicator (entirely or partly), the Human Development Report Office calculates it by converting GNI per capita from current to constant terms using two steps. First, the value of GNI per capita in current terms is converted into PPP terms for the base year (2011). Second, a time series of GNI per capita in constant 2011 PPP terms is constructed by applying the real growth rates to the GNI per capita in PPP terms for the base year. The real growth rate is implied by the ratio of the nominal growth of current GNI per capita in local currency terms to the GDP deflator.

To obtain the income value for 2016 for some countries, the International Monetary Fund (IMF)–projected real growth rates of GDP are applied to the most recent GNI values in constant PPP terms. The IMF-projected real growth rates are calculated based on local currency terms and constant prices rather than in PPP terms. This avoids mixing the effects of the PPP conversion with those of real growth of the economy.

Official PPP conversion rates are produced by the International Comparison Program, whose surveys periodically collect thousands of prices of matched goods and services in many countries. The last round of this exercise refers to 2011 and covered 199 countries.

Estimating missing values

For a small number of countries missing one of the four indicators, the Human Developmenr Report Office estimated the missing values using cross-country regression models. The details of the models used are available at http://hdr.undp.org.

In this Report expected years of schooling were estimated for the Bahamas, Bahrain, Dominica, Equatorial Guinea, the Federated States of Micronesia, Fiji, Haiti, Iraq, Papua New Guinea and the United Arab Emirates, and mean years of schooling were estimated for Antigua and Barbuda, Cabo Verde, Eritrea, Grenada, Guinea-Bissau, Kiribati, Palau, Saint Kitts and Nevis, Saint Vincent and the Grenadines, Seychelles and Turkmenistan.

Country groupings

This Report keeps the same cutoff points of the HDI for grouping countries that were introduced in the 2014 Report:

Very high human development	0.800 and above
High human development	0.700–0.799
Medium human development	0.550–0.699
Low human development	Below 0.550

Technical note 2. Inequality-adjusted Human Development Index

The Inequality-adjusted Human Development Index (IHDI) adjusts the Human Development Index (HDI) for inequality in the distribution of each dimension across the population. It is based on a distribution-sensitive class of composite indices proposed by Foster, Lopez-Calva and Szekely (2005), which draws on the Atkinson (1970) family of inequality measures. It is computed as a geometric mean of inequality-adjusted dimensional indices.

The IHDI accounts for inequalities in HDI dimensions by "discounting" each dimension's average value according to its level of inequality. The IHDI equals the HDI when there is no inequality across people but falls below the HDI as inequality rises. In this sense, the IHDI measures the level of human development when inequality is accounted for.

Data sources

Since the HDI relies on country-level aggregates such as national accounts for income, the IHDI must draw on additional sources of data to obtain insights into the distribution. The distributions are observed over different units—life expectancy is distributed across a hypothetical cohort, while years of schooling and income are distributed across individuals.

Inequality in the distribution of HDI dimensions is estimated for:

- Life expectancy, using data from abridged life tables provided by UNDESA (2015). This distribution is presented over age intervals (0–1, 1–5, 5–10, ... , 85+), with the mortality rates and average age at death specified for each interval.
- Mean years of schooling, using household surveys data harmonized in international databases, including the Luxembourg Income Study, Eurostat's European Union Survey of Income and Living Conditions, the World Bank's International Income Distribution Database, United Nations Children's Fund Multiple Indicator Cluster Surveys, ICF Macro Demographic and Health Surveys and the United Nations University's World Income Inequality Database.
- Disposable household income or consumption per capita using the above listed databases and household surveys—and for a few countries, income imputed based on an asset index matching methodology using household survey asset indices (Harttgen and Vollmer 2013).

A full account of data sources used for estimating inequality in 2015 is available at http://hdr.undp.org/en/statistics/ihdi/.

Steps to calculate the Inequality-adjusted Human Development Index

There are three steps to calculating the IHDI.

Step 1. Estimating inequality in the dimensions of the Human Development Index

The IHDI draws on the Atkinson (1970) family of inequality measures and sets the aversion parameter ε equal to 1.[1] In this case the inequality measure is $A = 1 - g/\mu$, where g is the geometric mean and μ is the arithmetic mean of the distribution. This can be written as:

$$A_x = 1 - \frac{\sqrt[n]{X_1 \dots X_n}}{\bar{X}} \qquad (1)$$

where $\{X_1, ..., X_n\}$ denotes the underlying distribution in the dimensions of interest. A_x is obtained for each variable (life expectancy, mean years of schooling and disposable household income or consumption per capita).

The geometric mean in equation 1 does not allow zero values. For mean years of schooling one year is added to all valid observations to compute the inequality. Income per capita outliers—extremely high incomes as well as negative and zero incomes—were dealt with by truncating the top 0.5 percentile of the distribution to reduce the influence of extremely high incomes and by replacing the negative and zero incomes with the minimum value of the bottom 0.5 percentile of the distribution of positive incomes. Sensitivity analysis of the IHDI is given in Kovacevic (2010).

Step 2. Adjusting the dimension indices for inequality

The inequality-adjusted dimension indices are obtained from the HDI dimension indices, I_x, by multiplying them by $(1 - A_x)$, where A_x, defined by equation 1, is the corresponding Atkinson measure:

$$I_x^* = (1 - A_x) \cdot I_x .$$

The inequality-adjusted income index, I^*_{income}, is based on the index of logged income values, I_{income^*} and inequality in income distribution computed using income in levels. This enables the IHDI to account for the full effect of income inequality.

Step 3. Combining the dimension indices to calculate the Inequality-adjusted Human Development Index

The IHDI is the geometric mean of the three dimension indices adjusted for inequality:

$$IHDI = (I^*_{Health} \cdot I^*_{Education} \cdot I^*_{Income})^{1/3} =$$

$$[(1 - A_{Health}) \cdot (1 - A_{Education}) \cdot (1 - A_{Income})]^{1/3} \cdot HDI.$$

The loss in the Human Development Index due to inequality is:

$$Loss = 1 - [(1 - A_{Health}) \cdot (1 - A_{Education}) \cdot (1 - A_{Income})]^{1/3}.$$

Coefficient of human inequality

An unweighted average of inequalities in health, education and income is denoted as the coefficient of human inequality. It averages these inequalities using the arithmetic mean:

$$Coefficient\ of\ human\ inequality = \frac{A_{Health} + A_{Education} + A_{Income}}{3}.$$

When all inequalities in dimensions are of a similar magnitude the coefficient of human inequality and the loss in HDI differ negligibly. When inequalities differ in magnitude, the loss in HDI tends to be higher than the coefficient of human inequality.

countries with medium-high equality in HDI achievements between women and men and are classified as group 2. Countries with absolute deviation from gender parity of 5–7.5 percent are considered countries with medium equality in HDI achieve-ments between women and men and are classified as group 3. Countries with absolute deviation from gender parity of 7.5–10 percent are considered countries with medium-low equality in HDI achievements between women and men and are classified as group 4. Countries with absolute deviation from gender parity of more than 10 percent are considered countries with low equality in HDI achievements between women and men and are classified as group 5.