PROBLEM SET 1 SOCIAL WELFARE
Inequality and Social Welfare

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OBS: The more * the questions presents the less important they are. Meaning question without any * are the most important ones and so on.

Exercise I

1) ***A population is divided into four groups, each one with four individuals. The individual incomes are:

Group 1	Group 2	Group 3	Group 4
1	1	2	4
1	1	2	4
2	2	4	8
4	4	8	16

Calculate the two Theil measures of inequality, verifying that the between and the within group components are the same for both measures.

2) **The individual incomes for three groups are given. In group 1, there are six individuals with incomes $x_{11} = x_{12} = 0,5$; $x_{13} = x_{14} = x_{15} = 1$ e $x_{16} = 8$. In group 2, there are five individuals with incomes $x_{21} = x_{22} = x_{23} = x_{24} = 1$ e $x_{25} = 16$. Group 3 has only three individuals and their incomes are $x_{31} = x_{32} = x_{33} = 16$. The three groups together constitute a total population of 14 individuals. Calculate the mean, median, mode, amplitude and variance of the income taking into account the 14 individuals. Calculate the Theil-T index related to the inequality in each group, the index related to global inequality and its within and between groups components. Do the same for the Theil-L index. Do the same for the Gini index.

3) ** Consider two populations divided into three stratum each. In population A, the 40% poorest have 10% of total income, the 40% of the middle have 40% and the 20% richest have 50%. In population B, the three stratum (40% poorest, 40% of the middle and 20% richest) have 20%, 20% and 60% of total income, respectively. We suppose there is no inequality within each stratum. Calculate the Gini index for each one of the two populations. Do the same for the Theil-T index and for the Theil-L index. Based on these results, verify that in each one of the two populations the income distribution is more unequal. Comment the results taking into account the Lorenz curve for each population.

4) Answer true or false and comment. The dual permits to compare different measures of inequality in the same scales and study the average sensibility of inequality to income transfers.

5) IBGE (National Bureau of Statistics and Geography) has recently launched the dual of the Theil index taking into account the working population with positive income using data from PNADs 2002 and 2003. Calculate the relative evolution (%) of inequality using the dual of the Theil-T index taking into account the active age population (therefore PIA – 15 to 65 years old).

Variation %

2003/2002	2002	2003	
-1.42%	0.563	0.555	**Dual of the Theil-T index for individuals with positive income**
2.43%	0.507	0.519	**% of PIA with zero income**
			Dual of the Theil-T index for the PIA

Calculate the Theil-T index for PNAD 2003

6) Answer true or false and comment. The Gini index for permanent income should be lower than for current income

    i.   We present the mean and the inequality of per capita income using the Gini index of a hipothetical country before and after a socialist revolution. Calculate the evolution of the well being in this society taking into account the function proposed by Sen.

Before Revolution	After Revolution	
300	250	**Mean Income**
0,6	0,5	**Gini**

7) Answer true or false and comment. The extension of temporal variability of observed income always influence inequality of annual incomes keeping constant the present value of the income earned during the life cycle.

8) Write down the formula and discuss possible problems of the following indicators:

- Sen's Social Welfare Function

- Variance of the logs as a measure of inequality

9) Write down the two alternative formulas and explain the logic and the intuition behind the Gini index.

10) According to the empirical evidence seen in class, the influence of the attribute in the decomposition of the Theil-T index is bigger when we measure:

    i)   The race attribute using household per capita income or individual labor income?

    ii)   The gender attribute using individual total labor income or labor income normalized per hour?

Exercise II

1) Calculate the inequality index seen in the course (Theil-T, Theil-L, Gini and the duals) according to the following sample of incomes: {1, 1, 2, 6, 30}

If we add one person with zero income to the sample, how do the indexes change?

2) Suppose that per capita income of household A, composed of only one individual, is 8. Suppose also that there is only another household in the economy, with incomes {1, 1, 2, 6, 30}. Calculate the level of inequality according to the following concepts:

  i)   Household per capita income between households

  ii)   Household per capita income between individuals

  iii)   Calculate the inequality component of individual income between groups of households (i.e., A and B). Assume now that income of household A is 7. Recalculate it.

  iv)   Suppose there is no socialization of incomes inside the households. How much of total income inequality is going to be underestimated taking into account both scenarios?

3) Write down the formulas and compare advantages and disadvantages of the Theil-T and Gini inequality indexes.

4) What is the meaning and the importance of the Principle of Transfers (Pigou-Dalton) in the specification of a Social Welfare Function?

5) Define and illustrate the concept of Lorenz dominance.

Exercise III

1) Empirical Estimates

A) For the model $\ln Y_i = a + b_2 X_i + u_i$, we have the following estimate

$$\hat{\ln} Y_i = 0.8972 + 0.1543 X_i$$

$$ep \quad (0.01768) \quad (0.0497)$$

$$R^2 = 0.4456$$

where
Y_i = Income from the main activity; X_i = Years of Study; ep are the standard errors of the estimates

Interpret the slope coefficient (give its formula), its significance and the R^2 of the regression.

2) Using the regression above and the Theil-T index, discuss and explain the logic of the role of education in the determination of the labor income inequality in Brazil.

Other exercises IV: (Handout 1B)
1 (5.)
Taking into account the following social welfare function, discuss how to incorporate the Principle of Transfers in the measures of inequality. What would be the case for the Gini and Atkinson's (with *Epsilon* = 1*)* measures?

$$W = u(x^*) = \int_0^\infty u(x)w(x)f(x)dx$$

2 (7.) Calculate the Theil-T index between groups by gender for per capita individual income using the following data. Interpret.

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Per Capita

Gender	Income	Population
Male	806.54	91507992
Female	468.31	96686391
Total	630.25	188194383

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3 (8) Explain how the following dimensions affect the measurement of inequality and its decomposition between and within groups:

a) Use disaggregated income between individuals from the same household and calculate the Theil and Gini index between households versus between individuals.

b) How to incorporate the issue of the temporal income variability at individual level on inequality measures. Remember in Brazil income is measured at a monthly basis.

 4 (9) Empirical

Consider the labor decomposition of individual income from different classical sources:

15 a 65 anos - Total								
População Total								
Categoria	**Ano**	**Renda de Todas as Fontes** =	**Renda de Todas as Fontes / Renda de Todos os Trabalhos** x	**Salário-Hora por Anos de Estudo dos Ocupados** x	**Anos de Estudo dos Ocupados** x	**Horas Trabalhadas** x	**Taxa de Ocupação na PEA** x	**Taxa de Participaçã o no Mercado de Trabalho**
Total	2009	806,56	1,1703	3,08	8,64	42,107	0,833	0,739
	2003	642,65	1,1874	2,82	7,619	43,468	0,803	0,721
	Taxa de Variação Anual (%)	3,86	-0,24	1,48	2,12	-0,53	0,61	0,41
		Tot Income	Income from all sources / Income from all jobs	Hourly-wage by education	Education	Work ing time	Occupied in EAF	EAP in AAP Participation

a. What is the level of unemployment in the PIA (Active Age Population -15 to 65 years)?

b. What is the fraction of the growth of the mean <u>labor</u> income in the PIA that is explained by the rise in occupation?

c. If we assume a 0.5% per year growth of the PIA as a result of the recent demographic transition, what should be the growth of income from all sources?

d. Compare the impacts in total income of the demographic bonus with the impacts of the rise in average years of schooling of the occupied (educational bonus).